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USE OF FORMULAS IN MECHANICS

APPLICATIONS TO ENGINEERING PROBLEMS LEVERS—STRENGTH OF BEAMS

THIRD REVISED EDITION

MACHINERY'S REFERENCE BOOK NO. 19
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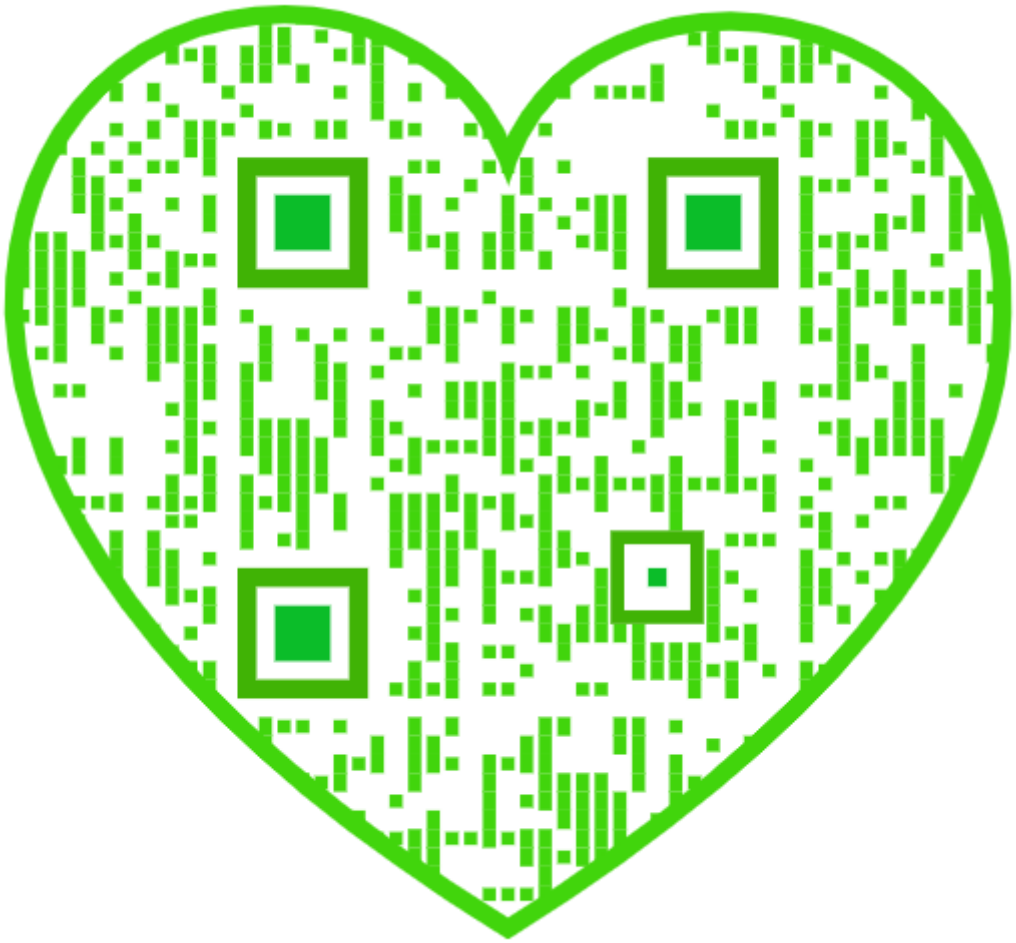

MACHINERY'S REFERENCE SERIES

EACH NUMBER IS A UNIT IN A SERIES ON ELECTRICAL AND STEAM ENGINEERING DRAWING AND MACHINE DESIGN AND SHOP PRACTICE

NUMBER 19

USE OF FORMULAS IN MECHANICS

THIRD REVISED EDITION

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Students whose knowledge of elementary arithmetic and its application to simple problems is too limited for intelligent study of this treatise, are advised to first study MACHINERY'S Jig Sheets 5A to 15A, inclusive, Common Fractions and Decimals; MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist; and No. 52, Advanced Shop Arithmetic for the Machinist.

In preparing the second edition of this book, the chapter on graphical methods of solving problems, contained in the first edition, was omitted, and in its place a chapter containing solutions of twenty-four mechanical problems selected from many different fields of mechanical engineering, were introduced. This substitution, it is believed, greatly enhanced the value of the book, and met with the approval of readers especially interested in the use of formulas in mechanics. In the present—the third—edition, this feature has, therefore, been retained.

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CHAPTER I

GENERAL REMARKS ON SELF-EDUCATION AND THE USE OF FORMULAS*

There are several ways of obtaining an education: The easiest and, until recent years, the usual way is to begin at the age of seven and continue steadily at school till the age of twenty-four, at father's expense. It is a fortunate fact that education is by no means unattainable otherwise; indeed many of the greatest and most widely useful educations the world has known have been obtained almost without a look at the inside of a school. A second method, quite modern, is the correspondence school—most excellent in many respects, yet not completing the available ways of obtaining an education. The final method is that of self-education. Nearly every successful man in engineering must necessarily obtain a very large share of his education in this manner, no matter what his general educational facilities have been; and it is for the purpose of explaining the possibilities of this method, and to plant the seed of self-help, that this and the following chapters have been written. They are divided into five heads dealing with the following subjects:

1. Present introduction, explaining general methods to be followed, and the principles of the use of formulas.
2. Examples of the use of formulas in mechanics.
3. The application of formulas to the solution of problems involving the principles of levers and moments, showing the simplicity of the form and application of the formulas.
4. The application of formulas in finding the center of gravity of geometrical figures.
5. The elements of the theory of the strength of materials, and the use of formulas in calculations of strength of beams.

It is the aim of these chapters to start the ambitious young man of sufficient grit upon a path which, if rightly followed, will in the future surely place him on par with those more fortunate men of his age who have enjoyed a college education, and to leave him in a position to continue to read and study and to understand the technical discussion and articles on design which appear in the technical press.

Engineering education does not consist in knowing things mechanical—far from it. It consists largely in knowing where to find technical literature upon any given subject when it is wanted, and knowing how to read it when it is found. Therefore, the first thing needed by our student is a place to store his newly acquired knowledge, aside from his head. The first attempt in this line of the author of this chapter, was a book having black canvas covers and a flexible back. Tapes were provided to lace in the leaves, which were made of fairly

* MACHINERY, October, 1905.

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heavy cardboard, perforated for the tapes, and having a flexible strip along the perforated edge to enable the leaves to turn back properly. Twenty-six alphabet leaves were made similar to those in dictionaries and memorandum books, and a supply of extra leaves kept on hand.

Clippings from papers and catalogues were pasted on blank leaves and inserted under the proper letter, also notes and formulas received from others were written in, making the book a record of past work and study. The book, finally becoming too large to be convenient and too small to hold everything to be preserved, gave way to the card index and filing case.*

Having provided a systematic way to file our clippings, we are ready to consider the sources of the same. First subscribe for one or two of the leading technical journals devoted to your line of work. Make a practice of sending for catalogues of machinery manufacturers, and file them in the filing box. Many catalogues present, besides the goods manufactured, tables and data of value. If you can clip out these tables and file them in the card index without destroying the catalogue, do so; if not, make an entry in the card index to show where they may be found, before filing the catalogue. Always write your name in the catalogues, for as the file grows, you will find demands upon it from others, and this will aid in keeping the file intact. Remember that a catalogue received implies confidence on the part of the sender that it will eventually prove of use to him by bringing his goods before possible purchasers, and for this reason, as well as for your own convenience, all catalogues received should be listed and filed.

Duplicate clippings, such as tables, may often be exchanged with others, and thus our files are enlarged. This is not meant to encourage a mere mania for collecting—far from it. We should so study all data filed as to understand it at the time, and if found difficult, make such notes as will readily recall the study to our minds in the future.

Mathematical Signs and Expressions

The first thing to be done in preparation for study, and for reading the technical papers, is to become familiar with the *engineering language*. The *spoken engineering language* is of course the native tongue of the country, with, however, plenty of new words to master; but the *written engineering language* consists very largely of symbols, so like those of higher mathematics in appearance as often to discourage the beginner from further efforts. In the *written engineering language*, rules, instead of being written in the native tongue, are expressed by combinations of these symbols, and when so expressed are called formulas.

Now, the mathematician, when deriving a formula, uses the same symbols as the engineer when writing a formula, and if we accept the work of the mathematician as correct, we need pay no attention to the use of these symbols in deriving formulas, but give our attention to learning to read the symbolic language of the engineer with sufficient

* See MACHINERY'S Reference Series No. 2, Drafting-Room Practice, second edition, page 44: Card Index for the Draftsman's Individual Records.

ease to enable us to follow the operations called for by any formula we may wish to use.

The following table exhibits in the first column the symbols most frequently met with; in the second column the arithmetical equivalent of the symbols is given, assuming that $a = 2$ and $b = 4$; in the third column the symbols are expressed in English to give the proper method of reading the symbols.

TABLE 1. COMMON MATHEMATICAL SIGNS

$a = 2 \quad b = 4$		a equals 2 b equals 4
$a + b = c$	$2 + 4 = 6$	a plus b equals c
$b - a = d$	$4 - 2 = 2$	b minus a equals d
$a \times b = e$ $a.b = e$ $ab = e$	$2 \times 4 = 8$	a times b equals e, or ab equals e
$a\,(a + b) = f$	$2 \times 6 = 12$	a times a plus b equals f
$b \div a = h$ $\frac{b}{a} = h$	$\frac{4}{2} = 2$	b divided by a equals h, or b over a equals h
$a < b$	$2 < 4$	a is less than b
$b > a$	$4 > 2$	b is greater than a
$b : a :: f : c$ $\frac{b}{a} = \frac{f}{c}$	$\frac{4}{2} = \frac{12}{6}$	b is to a as f is to c b divided by a equals f divided by c b over a equals f over c
$a^2 = b$	$2 \times 2 = 4$	a square equals b*
$b^3 = k$	$4 \times 4 \times 4 = 64$	b cube equals k
$\sqrt{b} = a$	$\sqrt{4} = 2$	square root of b equals a
$\sqrt[3]{e} = a$	$\sqrt[3]{8} = 2$	cube root of e equals a

Examples of Formulas

Let us now take the simple case of finding the area of a circle whose diameter we know. Expressed in English the rule is: Multiply the diameter by itself, then multiply the resulting product by 0.7854. The result is the area of the circle. If the diameter is expressed in inches, the area will be expressed in square inches. The corresponding mathematical expression is

$$A = 0.7854\,d^2 \qquad (1)$$

where A = the area in square inches,

d = the diameter in inches.

Note that d^2 simply means $d \times d$.

Now, to solve this expression for a particular case, suppose we wish to know the area of a circle nine inches in diameter. We simply substitute for d^2 its numerical value, and perform the indicated operation, thus:

$$A = 0.7854 \times 9 \times 9 = 0.7854 \times 81 = 63.617 \text{ square inches.}$$

* For a more complete explanation of the meaning of square and square root, and cube and cube root, see MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, or MACHINERY'S Jig Sheets No. 19A, Square Root, and No. 20A. Cube Root.

Take as another example the formula for the indicated horse-power of an engine:

$$H.P. = \frac{PLAN}{33{,}000} \qquad (2)$$

where P = the mean effective pressure in pounds per square inch,
L = the length of stroke in feet,
A = the area of the piston in square inches,
N = the number of strokes per minute.

Note that $PLAN$ simply means $P \times L \times A \times N$.*

The whole information as to how to determine the indicated horse-power of an engine is given in a very small space in the formula, while to write the same in English would require considerable of the space at our disposal.

Take the case of an 8 × 10-inch engine running at 100 revolutions per minute under 125 pounds mean effective pressure; here we have:

$P = 125$ pounds,

$$L = \frac{10 \text{ inches}}{12} = 0.833 \text{ feet},$$

$A = 0.7854 \times 8 \times 8 = 50.26$ square inches,

$N = 100$ rev. per min. $\times 2 = 200$.

Then,

$$H.P. = \frac{125 \times 0.833 \times 50.26 \times 200}{33{,}000} = 31.7$$

Right-angled Triangles

In right-angled triangles,† if we call the side opposite the right angle a, and the sides forming the right angle b and c, then the following formula expresses the relationship between the three sides:

$$a = \sqrt{b^2 + c^2} \qquad (3)$$

Assume, for example, that in a right-angled triangle one of the sides forming the right angle is 8 inches long, and the other side forming the right angle is 6 inches. What is the length of the side opposite the right angle?

If we insert the given dimensions in the formula above, we have:

$$a = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10.$$

The side opposite the right angle, thus, is 10 inches long.

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* See MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, or MACHINERY'S Jig Sheet No. 16A, Use of Formulas.

† See MACHINERY'S Jig Sheet No. 21A, Squares, Rectangles, Triangles, etc. For a more complete treatment of the right-angled triangle see MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, and No. 54, Solution of Triangles.

CHAPTER II

THE USE OF FORMULAS IN MECHANICS

The use of formulas for solving problems in mechanics can best be made clear by actual examples. In the present chapter, therefore, a number of problems have been solved, showing the methods employed, and the manner in which the formulas taken from hand books and reference works are used.

Problem 1.—A metal ball falls from the top of a tower 300 feet high. How long a time will be required before it reaches the ground?

The formula by means of which this problem is solved is:*

$$t = \sqrt{\frac{2h}{g}} \qquad (4)$$

in which t = time in seconds,

h = height in feet,

g = acceleration due to gravity = 32.16 feet.

Inserting the known values of h and g in the formula, we have:

$$t = \sqrt{\frac{2 \times 300}{32.16}} = \sqrt{18.66} = 4.32 \text{ seconds.}$$

Problem 2.—What is the velocity of the ball in the previous example when it reaches the ground?

The formula for finding the velocity is:

$$v = \sqrt{2gh} \qquad (5)$$

in which v = velocity in feet per second, and h and g denote the same quantities as in Problem 1. Inserting the values of g and h in the formula, we have:

$$v = \sqrt{2 \times 32.16 \times 300} = \sqrt{19{,}296} = 139 \text{ feet, nearly.}$$

Problem 3.—A projectile is fired from a 12-inch gun vertically into the air. It strikes the ground, coming down, exactly 1 minute and 40 seconds after it left the muzzle. Disregarding air resistance, what height did the projectile reach? What was its velocity when leaving the muzzle? And what is the energy of the projectile when it strikes the ground, if its weight is assumed to be 600 pounds?

The time required for the projectile to reach its greatest height is one-half of the total time for the upward and downward journey. Thus, in 50 seconds, the projectile has reached the point where its velocity is zero, and where it begins to fall. The formula for finding the height reached is:

$$h = \frac{gt^2}{2} \qquad (6)$$

* See MACHINERY'S Reference Series No. 5, First Principles of Theoretical Mechanics, page 34, second edition.

in which h, g and t denote the same quantities as in Problem 1. Inserting the known values, we have:

$$h = \frac{32.16 \times 50^2}{2} = \frac{32.16 \times 2{,}500}{2} = 40{,}200 \text{ feet,}$$

$$\text{or } \frac{40{,}200}{5{,}280} = 7.6 \text{ miles, approximately.}$$

The velocity of the projectile when leaving the muzzle is the same as the velocity acquired when again reaching the ground. This velocity is found by the formula:

$$v = gt = 32.16 \times 50 = 1{,}608 \text{ feet per second.} \qquad (7)$$

The energy of the projectile when it strikes the ground equals its weight multiplied by the distance through which it has fallen. If W = weight, and E = energy, we have:

$$E = W \times h = 600 \times 40{,}200 = 24{,}120{,}000 \text{ foot-pounds.} \qquad (8)$$

Another formula for the energy is as follows:

$$E = \frac{Wv^2}{2g}. \qquad (9)$$

This formula, with the values of W, v and g inserted, will, of course, give the same result.

$$E = \frac{600 \times 1{,}608^2}{2 \times 32.16} = \frac{600 \times 2{,}585{,}664}{2 \times 32.16} = 24{,}120{,}000 \text{ foot-pounds.}$$

If, upon reaching the ground, the projectile buries itself to a depth of 8 feet, what is the average force of the blow with which it strikes the ground? The average force of the blow equals the energy divided by the distance d in which it is used up, plus the weight of the projectile, or if F = average force of blow:

$$F = \frac{E}{d} + W = \frac{24{,}120{,}000}{8} + 600 = 3{,}015{,}600 \text{ pounds.} \qquad (10)$$

Problem 4.—A drop hammer weighing 300 pounds falls through a distance of 3 feet. What is the stored or kinetic energy of the hammer when it strikes the work, and what is the average force with which it delivers the blow, if, on striking the work, it compresses it ½ inch?

From Formula (8) given in Problem 3, we have:

$$E = W \times h = 300 \times 3 = 900 \text{ foot-pounds.}$$

The distance d in which this energy is used up equals ½ inch or $\frac{1}{2} \div 12 = 0.04$ foot. Therefore, from Formula (10) the average force is:

$$F = \frac{E}{d} + W = \frac{900}{0.04} + 300 = 22{,}500 + 300 = 22{,}800 \text{ pounds.}$$

Problem 5.—Find the stress in the rim of a fly-wheel, 5 feet mean diameter, made of cast iron, the rim being 2 inches wide by 4 inches deep, if the fly-wheel rotates at a velocity of 200 revolutions per minute.

The formula for the stress in the rim is:*

$$S = 0.00005427\ WRr^2 \qquad (11)$$

in which S = stress in pounds on the rim section,
W = weight of rim in pounds,
R = mean radius in feet, and
r = revolutions per minute.

We know that the mean diameter of the fly-wheel is 5 feet; therefore, $R = 2.5$ feet; r is given as 200; but we must find the value of W before we can apply Formula (11).

The weight W of the rim equals its volume or content in cubic inches multiplied by the weight of cast iron per one cubic inch. The volume of the rim equals the cross-sectional area of the rim multiplied by the circumference of the circle having for radius the mean radius of the flywheel; expressed as a formula:

$$V = 2\,R \times 3.1416 \times a \times b.$$

in which V = the volume of the rim, in cubic inches, R = the mean radius, in inches, a = the width, and b = the depth of the rim, in inches. Substituting the values in this formula, we have:

$$V = 2 \times 30 \times 3.1416 \times 2 \times 4 = 1{,}508 \text{ cubic inches.}$$

One cubic inch of cast iron weighs 0.26 pound. The weight of the rim then is:

$$W = 1{,}508 \times 0.26 = 392 \text{ pounds.}$$

We can now substitute the values in Formula (11):

$$S = 0.00005427 \times 392 \times 2.5 \times 200^2 = 2{,}127 \text{ pounds.}$$

The multiplication above can be carried out by the use of logarithms as follows:†

$$\begin{aligned} \log 0.00005427 &= \bar{5}.73456 \\ \log 392 &= 2.59329 \\ \log 2.5 &= 0.39794 \\ 2 \times \log 200 &= 4.60206 \\ \hline \log S &= 3.32785 \end{aligned}$$

Hence $S = 2{,}127$ pounds.

Problem 6.—The cylinder of a steam engine is 16 inches in diameter, and the length of the piston stroke 20 inches. The mean effective pressure of the steam on the piston is 110 pounds per square inch, and the number of revolutions per minute of the engine fly-wheel is 80. What is the power of the engine in indicated horse-power?

The formula for the horse-power of engine has been given in Chapter I, page 6:

$$H.P. = \frac{PLAN}{33{,}000} \qquad (2)$$

in which P = mean effective pressure in pounds per square inch,

* See MACHINERY'S Reference Series No. 40, Fly-Wheels, page 19, first edition.
† See MACHINERY'S Reference Series No. 53, Use of Logarithms and Logarithmic Tables.

L = length of stroke in feet,
A = area of piston in square inches,
N = number of strokes of piston per minute.

In the given problem $P = 110$; L (in feet) $= \frac{20}{12} = 1\,2/3$; A, the area of the piston in square inches $= 16^2 \times 0.7854 = 256 \times 0.7854 = 201.06$; and N, the number of strokes of piston per minute $= 2 \times$ revolutions of fly-wheel $= 2 \times 80 = 160$. Substituting these values in the formula, we have:

$$H.P. = \frac{110 \times 1\,2/3 \times 201.06 \times 160}{33{,}000} = 178.72.$$

Problem 7.—It is required to determine the diameter of cylinder and length of stroke of a steam engine to deliver 150 horse-power. The mean steam pressure is 75 pounds; the number of strokes per minute is 120. The length of the stroke is to be 1.4 times the diameter of the cylinder.

First insert in the horse-power Formula (2) the known values:

$$150 = \frac{75 \times L \times A \times 120}{33{,}000} = \frac{3 \times L \times A}{11}.$$

The last expression is found by cancellation.

Assume now that the diameter of the cylinder in inches equals D. Then $L = \frac{1.4\,D}{12} = 0.117\,D$, according to the requirements in the problem; the divisor 12 is introduced to change the inches to feet, L being in feet in the horse-power formula. The area $A = D^2 \times 0.7854$. If we insert these values in the last expression in our formula, we have:

$$150 = \frac{3 \times 0.117\,D \times 0.7854\,D^2}{11} = \frac{0.2757\,D^3}{11}$$

$$0.2757\,D^3 = 150 \times 11 = 1{,}650$$

$$D^3 = \frac{1{,}650}{0.2757}\,;\quad D = \sqrt[3]{\frac{1{,}650}{0.2757}} = \sqrt[3]{5984.8} = 18.15$$

The diameter of the cylinder, thus, should be about 18¼ inches, and the length of the stroke $18.15 \times 1.4 = 25.41$, or, say, 25½ inches.

Problem 8.—Find the horse-power required for compressing 10 cubic feet of air per second from 1 to 12 atmospheres, including the work of expulsion from the cylinder. Frictional and other losses are disregarded.

The formula for the work, W, in foot-pounds, required for compression and expulsion of 1 cubic foot of air from p_1 to p_n atmospheres is:

$$W = 3.463\,p_1 \left[\left(\frac{p_n}{p_1}\right)^{0.29} - 1\right] \times 14.7 \times 144 \qquad (12)$$

In the given problem $p_1 = 1$; $p_n = 12$; and as we are to compress 10 cubic feet instead of one, we must multiply the whole expression by 10. Thus:

$$W = 3.463 \times 1 \times \left[\left(\frac{12}{1}\right)^{0.29} - 1\right] \times 14.7 \times 144 \times 10$$

$$= 3.463 \times (12^{0.29} - 1) \times 14.7 \times 144 \times 10.$$

The value of the expression $12^{0.29}$ can be found only by the use of logarithms.*

$$\log 12 = 1.07918.$$
$$\log 12^{0.29} = 1.07918 \times 0.29 = 0.31296.$$
$$12^{0.29} = 2.056, \text{ and } 12^{0.29} - 1 = 1.056.$$

Hence:

$$W = 3.463 \times 1.056 \times 14.7 \times 144 \times 10 = 77{,}410 \text{ foot-pounds.}$$

This last result may be found by ordinary multiplication, or, more quickly, by logarithms as follows:

$$\log \cdot\ 3.463 = 0.53945$$
$$\log\ \ 1.056 = 0.02366$$
$$\log\ 14.7\ \ = 1.16732$$
$$\log 144\ \ = 2.15836$$
$$\log\ 10\ \ = 1.00000$$

$$\log W = 4.88879 \qquad W = 77{,}410.$$

As a horse-power equals 550 foot-pounds per second, the horse-power required for compressing 10 cubic feet of air from 1 to 12 atmospheres equals:

$$H.P. = \frac{77{,}410}{550} = 151 \text{ horse-power.}$$

Problem 9.—It is required to lift a weight weighing 1 ton by means of a screw having a lead of ½ inch. A lever passing through the head of the screw, and extending 4 feet out from the center, is provided at its outer end with a handle. How great a force must be applied at this handle to lift the required weight, friction being disregarded?

Let the weight to be lifted, in pounds, be W; the force applied at the end of the lever, F; the lead of the screw, l; and the length of the lever, in inches, r. The distance passed through by force F times this force must equal the distance weight W is lifted times the weight, or, expressed as a formula:

$$F \times 2r \times 3.1416 = W \times l. \qquad (13)$$

This formula is based on the fact that during one revolution of the screw and handle, force F acts through a distance equal to the circumference of the circle described by the handle, while the weight W is lifted an amount equal to the lead of the screw. If we insert the given values in the formula above, we have:

* See MACHINERY'S Reference Series No. 53, Use of Logarithms and Logarithmic Tables.

$$F \times 2 \times 48 \times 3.1416 = 2{,}000 \times \tfrac{1}{2}$$
$$F \times 301.59 = 1{,}000$$
$$F = \frac{1{,}000}{301.59} = 3.3 \text{ pounds.}$$

It will be seen that by the given arrangement a force of 3.3 pounds would be sufficient to lift a ton. Friction, however, has not been considered in this problem, and as the frictional resistance in machines using screws for conveying power is considerable, the actual force required would be a great deal more than 3.3 pounds.

Assume that is required to find the power if friction is considered. In this case we must know the diameter of the screw and the form of the thread. We will assume that the thread is square, and that the diameter of the screw is 3 inches. The depth of a ½-inch lead square thread is ¼ inch. The pitch diameter of the screw is, therefore, 3 — ¼ = 2¾ inches.

The formula for finding the force required at the end of the handle is:

$$Q = W \frac{f + \tan a}{1 - f \tan a} \times \frac{R}{r}$$

in which Q = force at end of handle, in pounds,
W = weight to be lifted = 2,000 pounds,
f = coefficient of friction,
a = angle of helix of the thread at the pitch diameter,
R = pitch radius of screw in inches = 1⅜ inch,
r = length of handle in inches = 48.

$$\tan a = \frac{\text{lead}}{3.1416 \times \text{pitch diam.}} = \frac{0.5}{3.1416 \times 2.75} = 0.058.$$

The coefficient of friction, f, may be assumed to be 0.15. If we now insert the known values in the formula, we have:

$$Q = 2{,}000 \times \frac{0.15 + 0.058}{1 - 0.15 \times 0.058} \times \frac{1.375}{48} = 12.02 \text{ pounds,}$$

or nearly four times as much as when friction was not considered.

Problem 10.—Determine the length of the main bearing of a large horizontal steam engine. The diameter of the crank-shaft is 10 inches, and the weight of the shaft, fly-wheel, crank-pin and other moving parts that may be supported by the bearings is 60,000 pounds. Assume that two-thirds of this weight, or 40,000 pounds, comes on the main bearing. The engine runs at 80 revolutions per minute.

The length of the main bearing of an engine may be found by the formula:*

$$L = \frac{W}{PK}\left(N + \frac{K}{D}\right) \qquad (14)$$

* See MACHINERY'S Reference Series No. 11, Bearings, page 11, first edition.

in which L = length of bearing in inches,
W = load on bearing in pounds,
P = maximum safe unit pressure on bearing at a very slow speed,
K = constant depending upon the method of oiling and care which the journal is likely to get,
N = number of revolutions per minute,
D = diameter of bearing in inches.

The safe unit pressure P for shaft bearings is 400 pounds; the factor K varies from 700 to 2,000. In this case, assume first-class care and drop-feed lubrication, in which case $K = 1{,}000$. The other quantities given are $W = 40{,}000$, $N = 80$, and $D = 10$.

Inserting these values in Formula (14), gives us:

$$L = \frac{40{,}000}{400 \times 1000}\left(80 + \frac{1000}{10}\right) = \frac{1}{10}(80 + 100) = 18 \text{ inches.}$$

Problem 11.—What is the carrying capacity of a helical spring having an outside diameter of 5 inches, made from ½-inch round steel? The tensile stress per square inch of section of spring must not exceed 80,000 pounds.

The formula for the carrying capacity of helical springs is:*

$$P = \frac{S\,d^3}{2.55\,D} \qquad (15)$$

in which P = safe carrying capacity,
S = safe tensile stress per square inch,
d = diameter of wire,
D = mean diameter of spring = outside diameter minus diameter of wire.

In the given problem $S = 80{,}000$; $d = ½$; and $D = 5 - ½ = 4½$. If these values are inserted in Formula (15) we have:

$$P = \frac{80{,}000 \times 0.5^3}{2.55 \times 4.5} = \frac{10{,}000}{11.475} = 871 \text{ pounds.}$$

Problem 12.—Find the weight of steam that will flow in one minute through a pipe 100 feet in length and 2 inches in diameter, if the initial pressure is 40 pounds (absolute) per square inch and the terminal or delivery pressure 35 pounds (absolute).

The formula for finding the weight of steam under the above conditions is:†

$$W = c\sqrt{\frac{w\,(P - P_1)\,d^5}{L}} \qquad (16)$$

in which W = pounds of steam per minute,
c = constant = 52.7 for a 2-inch pipe,

* See MACHINERY'S Data Sheet No. 22, July, 1903, Formulas for Coil Springs.
† See MACHINERY'S Data Sheet No. 109, March, 1909, Steam Pipe Sizes for Heating Systems.

w = weight per cubic foot of steam at initial pressure, in pounds,
P = initial pressure in pounds per square inch,
P_1 = terminal pressure in pounds per square inch,
d = diameter of pipe in inches,
L = length of pipe in feet.

In the present problem, $c = 52.7$; $w = 0.0972$ (obtained from tables in standard hand books); $P = 40$; $P_1 = 35$; $d = 2$; and $L = 100$. Inserting these values in Formula (16) gives:

$$W = 52.7\sqrt{\frac{0.0972 \times (40 - 35) \times 2^5}{100}} = 52.7\sqrt{0.1555} = 20.76 \text{ pounds.}$$

Problem 13.—Find the tractive power of a simple locomotive having 22-inch cylinder diameters, 26-inch stroke, a boiler pressure of 200 pounds, and 60-inch diameter driving wheels.

The formula for the tractive force of a locomotive is:*

$$T = \frac{0.85\,P\,d^2\,s}{D} \qquad (17)$$

in which T = tractive force in pounds,
P = boiler pressure in pounds per square inch,
d = diameter of cylinders in inches,
s = length of stroke in inches,
D = diameter of driving wheels.

Inserting the known values in Formula (17), gives:

$$T = \frac{0.85 \times 200 \times 22^2 \times 26}{60} = 35{,}655 \text{ pounds.}$$

Problem 14.—Find the diameter of the cylinders of a simple locomotive, having a tractive force of 30,000 pounds; length of stroke, 22 inches; diameter of driving wheels, 57 inches; and boiler pressure, 180 pounds.

The formula for the cylinder diameter is:†

$$d = \sqrt{\frac{T \times D}{P \times 0.85 \times s}} \qquad (18)$$

in which the letters denote the same quantities as in Formula (17).

If we insert the known values $T = 30{,}000$; $D = 57$; $P = 180$; and $s = 22$, in Formula (18), we have:

$$d = \sqrt{\frac{30{,}000 \times 57}{180 \times 0.85 \times 22}} = \sqrt{508.02} = 22.54 \text{ inches,}$$

or, approximately, 22½ inches diameter.

Problem 15.—Find the thickness of a cast iron cylinder to withstand a pressure of 1,000 pounds per square inch; the inside diameter of the cylinder is to be 10 inches, and the maximum allowable fiber stress per square inch 4,000 pounds.

* See MACHINERY'S Data Sheet No. 79, Constants for Calculating Tractive Force.
† See MACHINERY'S Reference Series No. 27, Locomotive Design, page 7.

The thickness is found by the following formula:*

$$t = \frac{D}{2}\left(\sqrt{\frac{S+P}{S-P}} - 1\right) \qquad (19)$$

in which t = thickness of cylinder wall in inches,
D = inside diameter of cylinder in inches,
P = working pressure in pounds per square inch,
S = allowable fiber stress in pounds per square inch.

Inserting the given values in Formula (19), we have:

$$t = \frac{10}{2}\left(\sqrt{\frac{4000+1000}{4000-1000}} - 1\right) = 5\left(\sqrt{1.667} - 1\right) =$$

$5\ (1.29 - 1) = 5 \times 0.29 = 1.45$, or say 1½ inch.

Problem 16.—How many cubic feet of air does a disk fan, 30 inches in diameter, deliver when running at a speed of 500 revolutions per minute?

The answer to this problem is found by the following formula:†

$$C = 0.6\,D\,R\,A, \qquad (20)$$

in which C = cubic feet of air delivered per minute,
D = diameter of fan in feet,
R = revolutions per minute,
A = area of fan in square feet.

In the given problem D, in feet $= \frac{30}{12} = 2.5$; $R = 500$; and $A = D^2 \times 0.7854 = 2.5^2 \times 0.7854 = 4.909$. Inserting these values in Formula (20), we have:

$$C = 0.6 \times 2.5 \times 500 \times 4.909 = 3{,}681.75 \text{ cubic feet.}$$

Problem 17.—What should be the weight of an 8-foot mean diameter fly-wheel, in pounds, for a two-cylinder, single-acting gas engine of 120 brake horse-power used in an electric lighting plant with continuous current generators, if the engine makes 300 revolutions per minute?

The following formula,‡ by Mr. R. E. Mathot, may be used for solving this problem:

$$P = K\frac{10.75\,N}{D^2\,a\,n^2} \qquad (21)$$

in which P = the weight of the rim, without arms or hub, in tons,
K = coefficient varying with the type of engine = 21,000 for a two-cylinder single-acting engine,
N = brake horse-power of engine,
D = mean diameter of fly-wheel, in feet,
a = amount of allowable variation = 1/50 for electric lighting by continuous current,
n = number of revolutions per minute.

* See MACHINERY'S Reference Series No. 17, Strength of Cylinders, page 21, first edition.
† See MACHINERY'S Reference Series No. 39, Fans, Ventilation and Heating, page 24.
‡ See MACHINERY'S Reference Series No. 40, Fly-Wheels, page 20, first edition.

In the given problem, where $K = 21{,}000$, $N = 120$, $D = 8$, $a = 0.02$, and $n = 300$, we have:

$$P = 21{,}000 \times \frac{10.75 \times 120}{8^3 \times 0.02 \times 300^2} = 0.78 \text{ ton.}$$

Expressed in pounds the weight of the rim equals $0.78 \times 2{,}000 = 1{,}560$ pounds.

Problem 18.—Find the thickness of the piston for a steam engine having a cylinder diameter of 20 inches and a length of stroke of 24 inches.

The following formula may be used for finding the thickness of the piston:*

$$T = \sqrt[4]{L \times D} \qquad (22)$$

in which T = thickness of piston in inches,
L = length of stroke in inches,
D = diameter of cylinder in inches.

Inserting the given values in this formula, we have:

$$T = \sqrt[4]{24 \times 20} = \sqrt[4]{480}.$$

The fourth root of 480 can be most easily found by logarithms.†

$$\log T = \frac{\log 480}{4}.$$

$\log 480 = 2.68124$; $2.68124 \div 4 = 0.67031$.
$\log T = 0.67031$; $T = 4.68$ inches.

Problem 19.—Find the average horse-power required for taking a chip in a lathe 5/16 inch deep with a feed of 5/32 inch per revolution. The material cut is a bar of 30-point carbon steel, 4 inches in diameter, and is turned at a speed of 40 revolutions per minute.

A formula for finding the horse-power for turning in a lathe, based upon the experiments of Hartig, is as follows:‡

$$H.P. = 0.035 \times 3.1416 \times D \times n \times d \times t \times 0.28 \times 60 \qquad (23)$$

in which $H.P.$ = horse-power required for turning,
D = mean diameter of piece turned,
n = revolutions per minute,
d = depth of cut,
t = thickness of chip = feed per revolution.

In the problem given, D = outside diameter minus depth of cut = $4 - 5/16 = 3\ 11/16$; $n = 40$; $d = 5/16$; and $t = 5/32$. If we insert these values in the given formula, we have:

$H.P. = 0.035 \times 3.1416 \times 3.6875 \times 40 \times 0.3125 \times 0.1562 \times 0.28 \times 60 = 13.3$.

Problem 20.—What horse-power may safely be transmitted by a 3 inches wide, machine-cut spur gear of 16-inch pitch diameter having 64 teeth, made of cast iron and running at a velocity of 120 revolutions per minute?

* See MACHINERY'S Data Sheet No. 120, Steam Engine Design.
† See MACHINERY'S Reference Series No. 53, Use of Logarithms and Logarithmic Tables.
‡ See MACHINERY'S Reference Series No. 16, Machine Tool Drives, page 29, first edition.

The formulas for the solution of this problem are as follows:*

$$V = 0.262\,D\,R \tag{24}$$

$$S = S_s \times \frac{600}{600 + V} \tag{25}$$

$$W = \frac{S\,F\,Y}{P} \tag{26}$$

$$H.P. = \frac{W\,V}{33{,}000} \tag{27}$$

in which V = velocity in feet per minute at pitch diameter,
D = pitch diameter in inches,
R = revolutions per minute,
S = allowable unit stress of material at given velocity,
S_s = allowable static unit stress of material,
W = maximum safe tangential load, in pounds, at pitch diameter,
Y = factor dependent upon pitch and form of tooth,
F = width of face of gear,
P = diametral pitch.
$H.P.$ = horse-power transmitted,

The known values to be inserted in the given formulas are $D = 16$, $R = 120$, S_s (for cast iron, assumed) $= 6{,}000$, $F = 3$; Y (for 64 teeth, standard form) $= 0.36$; and $P = 64 \div 16 = 4$. If we insert these values, as required, in the Formulas (24) to (27), and insert the values obtained in each formula in the next succeeding one, we have:

$$V = 0.262 \times 16 \times 120 = 503 \text{ feet.}$$

$$S = 6{,}000 \times \frac{600}{600 + 503} = 3{,}264 \text{ pounds per square inch.}$$

$$W = \frac{3{,}264 \times 3 \times 0.36}{4} = 881 \text{ pounds.}$$

$$H.P. = \frac{881 \times 503}{33{,}000} = 13.4 \text{ horse-power.}$$

Problem 21.—The initial absolute pressure of the steam in a steam engine cylinder is 120 pounds; the length of the stroke is 26 inches, the clearance 1½ inch, and the period of admission, measured from the beginning of the stroke, 8 inches. Find the mean effective pressure.

The mean effective pressure is found by the formula:

$$p = \frac{P\,(1 + \text{hyp. log } R)}{R} \tag{28}$$

in which p = mean effective pressure in pounds per square inch,
P = initial absolute pressure in pounds per square inch,

* See MACHINERY'S Reference Series No. 15, Spur Gearing, page 29, second edition.

R = ratio of expansion, which in turn is found from the formula:

$$R = \frac{L + C}{l + C} \qquad (29)$$

in which L = length of stroke in inches,
l = period of admission in inches,
C = clearance in inches.

The given values are $P = 120$; $L = 26$; $l = 8$; and $C = 1\frac{1}{2}$. By inserting the latter three values in Formula (29), we have:

$$R = \frac{26 + 1\frac{1}{2}}{8 + 1\frac{1}{2}} = \frac{27.5}{9.5} = 2.89.$$

If we now insert the value of P and the found value of R in Formula (28), we have:

$$p = \frac{120\ (1 + \text{hyp. log } 2.89)}{2.89}.$$

The hyperbolic logarithm (hyp. log.) must be found from tables giving its value.* The hyperbolic logarithm for 2.89 is 1.0613. Inserting this value in our formula, we have:

$$p = \frac{120\ (1 + 1.0613)}{2.89} = \frac{120 \times 2.0613}{2.89} = 85.6 \text{ lbs. per square inch.}$$

Problem 22.—It is required to pump 12 cubic feet of water per minute with a centrifugal pump, raising it 35 feet, 15 feet by suction and 20 by discharge pressure. What will be the diameter of suction and discharge pipe required?

According to a formula by Fink:

$$d = 0.36 \sqrt{\frac{Q}{\sqrt{2g\ (h + h_1)}}} \qquad (30)$$

in which Q = quantity of water, in cubic feet, pumped per minute,
g = acceleration due to gravity = 32.16,
h = height of suction in feet,
h_1 = height of discharge in feet,
d = diameter of suction and discharge pipe, in feet.

Inserting the known values in Formula (30) we have:

$$d = 0.36 \sqrt{\frac{12}{\sqrt{2 \times 32.16\ (15 + 20)}}} = 0.36 \sqrt{\frac{12}{47.45}} = 0.36 \times 0.5 = 0.18,$$

approximately.

A pipe 0.18 foot, or $2\frac{1}{8}$ inches, in diameter would be required.

Problem 23.—What is the average pressure on the tool when turning hard cast iron, taking a chip $\frac{1}{8}$ inch deep with 1/16 inch feed per revolution?

* See MACHINERY'S Reference Series No. 53, Use of Logarithms and Logarithmic Tables.

The formula given by F. W. Taylor for finding the pressure on the tool is:*

$$P = CD^{\frac{14}{15}} F^{\frac{3}{4}} \qquad (31)$$

in which P = average pressure on tool in pounds,

C = a constant = 69,000 for hard cast iron,

D = depth of cut in inches,

F = feed per revolution, in inches.

Inserting the known values in this formula, we have:

$$P = 69{,}000 \times 0.125^{\frac{14}{15}} \times 0.062^{\frac{3}{4}}$$

To find the values of the two last expressions in the product above, we must make use of logarithms.† The whole product is also most easily found by means of logarithms.

$$\log 0.125 = \bar{1}.09691; \quad \frac{14}{15} \times \bar{1}.09691 = \bar{1}.15712$$

$$\log 0.062 = \bar{2}.79239; \quad \frac{3}{4} \times \bar{2}.79239 = \bar{1}.09429$$

$$\log 69{,}000 = 4.83885$$

$$\log P = 3.09026$$

Hence, $P = 1{,}231$ pounds.

Problem 24.—Find the diameter of shaft required to transmit 60 horse-power at 300 revolutions per minute, if the maximum safe stress of the material of which the shaft is made is 10,000 pounds per square inch.

The formula for finding the diameter of shaft is:

$$d = \sqrt[3]{\frac{321{,}400 \times H.P.}{RS}} \qquad (32)$$

in which d = diameter of shaft in inches,

$H.P.$ = horse-power to be transmitted,

R = revolutions per minute,

S = safe shearing stress of material of which shaft is made.

If we insert the given values in Formula (32), we have:

$$d = \sqrt[3]{\frac{321{,}400 \times 60}{300 \times 10{,}000}} = \sqrt[3]{6.428} = 1.86 \text{ inch.}$$

The diameter of the shaft may, therefore, be made, say, 1⅞ inch diameter.

* See MACHINERY, June, 1907, engineering edition, page 568.

† See MACHINERY'S Reference Series No. 53, Use of Logarithms and Logarithmic Tables.

CHAPTER III

PRINCIPLE OF MOMENTS AS APPLIED TO THE LEVER*

The lever is the simplest element of a machine, and the principles of its action are of a simple nature. There is no reason why anyone who chooses to devote a little time to study should not be able to master these principles, and having done this, he will have gone a long way toward mastering the principles of all the elements that make up a machine.

Webster defines a lever as "a bar of metal, wood or other substance, used to exert a pressure or to sustain a weight, at one point at its length, by receiving a force or power at a second, and turning at a third on a fixed point called a fulcrum. It is of three kinds, according as either the fulcrum *F*, the weight *W*, or the power *P*, respectively, is situated between the other two." This is the usual definition of a lever as it is found in most books on mechanics and physics, and attention should be called to certain points about it that could easily lead a beginner astray and cause confusion at the outset. It is always best to start with a clear idea of a subject, so that there will be no uncertainty to begin with.

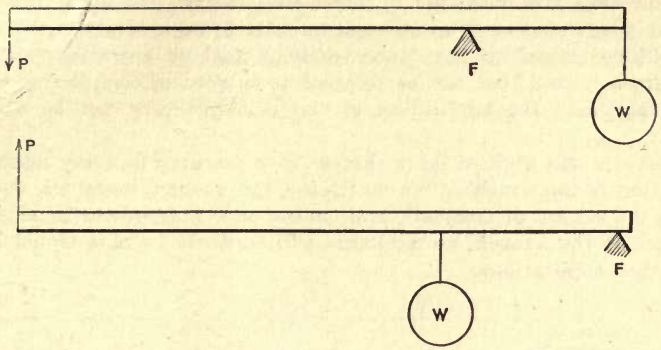
In Fig. 1 is a lever, in which, according to the definition, *W* is a weight acting at one point, *P* is the power or force acting at another point to raise the weight *W*, as indicated by the arrow, and *F* is the fulcrum on which the lever turns. That part of the lever between the weight and the fulcrum is called the "weight arm," and that part between the fulcrum and the power is called the "power arm." It will be noted that the fulcrum in Fig. 1 is located between the weight and power. In Figs. 2 and 3, however, are two levers in which the arrangement is different, the weight being placed between the power and fulcrum in Fig. 2, and the power placed between the weight and fulcrum in Fig. 3. These three figures illustrate the first, second, and third kinds of lever, as above defined.

The objections to this definition of the lever are, in the first place, the use of the word "power" for the force applied at the end of the lever to raise the weight. "Power" has a totally different meaning from "force," and takes into account not only force, but time and distance. A force is merely a push or pull, such as is exercised by the hand, and this is the kind of effort that is always required to raise a weight or overcome any other resistance. In the reference letters of the illustrations, therefore, we will let *P* stand for a push or a pull, as the case may be, instead of for the word "power." Hereafter, also, instead of calling the resistance to be overcome the "weight," we will

* MACHINERY, October and November, 1898.

call it the "resistance" and represent it by the letter *R*. A lever may have to overcome a number of resistances besides that of raising a weight, such as the resistance of friction, of a coiled spring, or of the pressure of steam, and the term "resistance" implies this better than the term "weight."

Finally, regarding the three kinds of levers mentioned above, there is no necessity for trying to separate levers into any number of classes, or for trying to remember to which class they belong in the solution of examples. All levers depend upon the same principles, which are simple



Figs. 1 and 2

and easily understood, and all that is necessary is to first master these principles without regard to the relative position of the applied force, the resistance, or the fulcrum.

The Moment of a Force

We have seen what is meant by the term "force," and the next thing to learn is what the moment of a force is. When a force acts at a point on a lever, that is, when that point is given a push or a pull, the tendency is to cause the lever to turn about its fulcrum. This tendency

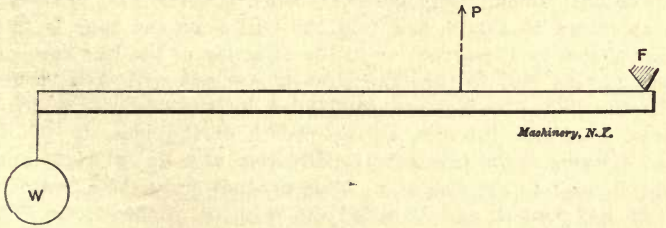


Fig. 3

depends first upon the strength of the force acting and second upon the perpendicular distance from the line of action of the force to the fulcrum. If either the strength of the push or pull exerted by the force, or the perpendicular distance of its line of action from the fulcrum, is changed, the tendency of the force to rotate the lever will be greater or less, as the case may be. The rotative effect of any force thus depends upon both the strength and the distance, and is measured

by their product, this product being called the moment of the force. The moment of force, therefore, is the measure of the turning effect of that force, and is found by multiplying the force by the perpendicular distance from its line of action to the fulcrum. If the force be measured in pounds and the distance in feet, the moment will be in foot-pounds; if the force be in pounds and the distance in inches, the moment will be inch-pounds; if the force be in tons and the distance in feet, the moment will be in foot-tons, etc. The foot-pounds measurement is the most commonly used, however.

This subject of moments is important—in fact, the most important in the whole subject of levers—and in order to fix it firmly in the mind, it will be helpful to have some common fact or operation that will illustrate it, and that can be referred to in solving complicated examples in which the application of the principle may not be entirely clear.

There is one kind of lever that is very familiar to every mechanic, and that is the wrench. We will select the wrench, therefore, to illustrate the subject of moments, and having once grasped the principle as applied to the wrench, no mechanic will be likely to have trouble with its other applications.

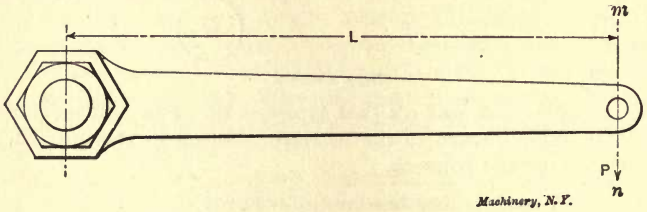


Fig. 4

Fig. 4 represents a box wrench, and, as is often done in work of a heavy character, a hole is punched in the outer end of the handle, into which a chain or rope can be hooked or fastened to assist in screwing the bolt or nut "home." Suppose the wrench is being used to screw up a nut, as shown in Fig. 4, and that the pull P on the rope is in the direction shown by the arrow, or in the direction of the line $m\,n$. The tendency of this pull to turn the wrench and nut will then be measured by the pull P in pounds, multiplied by the distance L in feet measured from the fulcrum at the center of the bolt, to the line $m\,n$, the distance being taken in the direction of a line at right angles or perpendicular to the line $m\,n$. This product gives the effect of the pull P in foot-pounds, and is called the moment of this force. Thus, if the pull P is 300 pounds, and the length L is 4 feet, the moment of the force P is $300 \times 4 = 1{,}200$ foot-pounds, and this is the measure of the turning effect of this force.

The reason why this is so will be evident if we consider another case shown in Fig. 5. Here the wrench has been placed in a new position, ready for another turn, and the pull P acts in the same direction as before, along the line $m\,n$. Now, anybody who has used a wrench knows that with the same pull a greater effect will be pro-

duced with the wrench as placed in Fig. 4 than as placed in Fig. 5, although in each case the hook is at the same distance (4 feet) from the fulcrum *F*. The direct distance, however, of the point of application of the force from the fulcrum does not necessarily have any influence on the effectiveness of this force in moving the lever. The only distance that can be considered is the perpendicular distance from the line along which the force acts to the fulcrum, and this distance is greater in Fig. 4 than in Fig. 5, and in the former the force of 300 pounds has a greater leverage than in the latter. In Fig. 5 the measure of the rotative effect is the pull *P*, which is 300 pounds, times the distance *L*, which in this case measures 2 feet, or $300 \times 2 = 600$ foot-pounds. The distance *L*, as before, is measured at right angles to

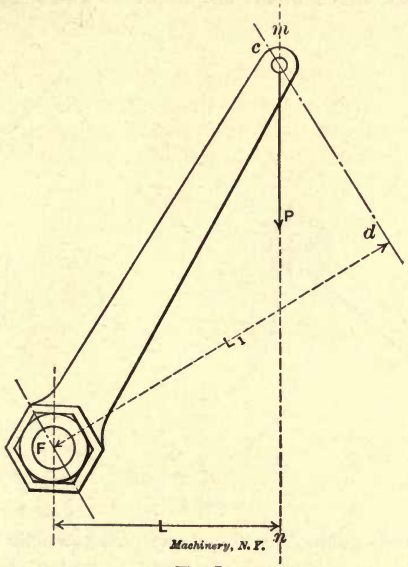


Fig. 5

the line *m n*, and if the rope had extended along the line *c d*, instead of the line *m n*, *L* would have been measured at right angles to the line *c d*, as indicated by the line L_1.

The True Lever Arm

The distance *L* in Figs. 4 and 5 is called the lever arm. Ordinarily the arm of a lever is understood to mean that part of the lever that lies between the fulcrum and the point where the force is applied, or between the fulcrum and the point where the resistance takes place; and such it is in a strict sense if the lever arm is straight and the force acts at right angles to the lever. But in Fig. 5 the true length of the lever arm is the distance *L*, and not the length of the handle of the wrench, because *L* is the effective length acting, in the position shown. *The true lever arm*, therefore, *is the perpendicular distance from the line of action of the force to the fulcrum.*

A familiar example of the moment of a force is to be had in the action of the foot in pedaling a bicycle. When the crank has passed the upper center, and the foot is ready for the downward push, it will require a much greater effort to drive the wheel ahead than when the crank is at right angles to the direction of the motion of the foot. The crank, of course, is of the same length whatever its position; but considered as a lever, the length of its arm varies from nothing at the upper center, to the full length of the crank at the extreme forward movement of the foot. The moment of the force exerted by the foot, therefore, gradually increases from nothing at the upper part of the stroke to the greatest amount at the forward position.

Still another illustration is to be had in the curved crank shown in Fig. 6. The crank turns about the point *F*, and a rod is attached at

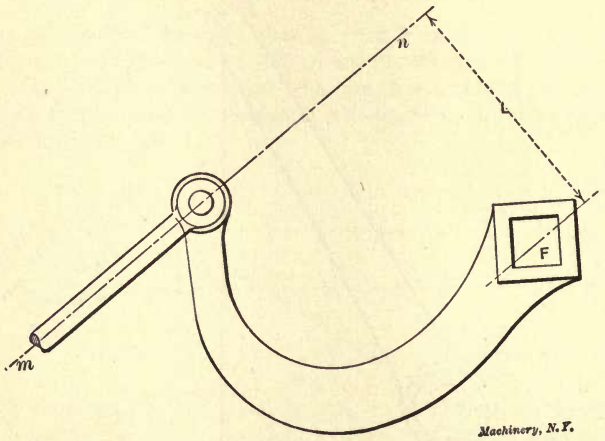


Fig. 6

the outer end which pushes in the direction shown by line *m n*. Drawing this dotted line *m n* through the point at which the push is applied and in the direction in which the push is exerted, we have *L*, which is drawn at right angles to *m n*, as the length of the lever arm, and the moment of the force is the length *L* multiplied by the force *P*.

The Principle of Moment

Thus far the illustrations that have been used have pertained to what might be called single-armed levers. We have considered only the forces acting without regard to the resistance that had to be overcome, and the levers themselves have been more of the nature of a crank than of a lever, though it is not always easy to make a distinction between the two. It is evident, however, that wherever a force is exerted, there must also be a resistance, as otherwise no initial force would be required to create motion. In the case of the wrench, the resistance was the friction between the threads of the bolt and nut acting at the end of a lever arm equal to the radius of the bolt; and

in the case of the bicycle crank, the resistance was at the rim of the bicycle wheel, the lever arm in this case being more complicated because of the sprockets and chain.

In Fig. 7 is shown a bell-crank lever pivoted at the fulcrum *F*. A pull *P* is exerted along the rod at the left, and this is balanced by another pull along the rod at the right, which acts as a resistance to the force *P*. To determine the relative rotative effects of the pull *P* and the resistance *R*, we must determine the moments of these two forces. To find the moment of *P*, draw a line *m n* through the point of the lever at which *P* takes effect, and in the direction of the line in which it acts. Then draw the line *L* from the fulcrum *F* and at right angles to the line *m n*. This will be the true lever arm, and the

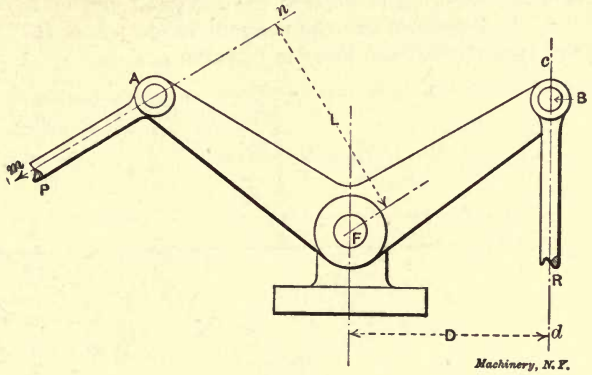


Fig. 7

moment of *P* will be the product of *P* and the length *L*. To find the moment of *R*, draw the line *c d* through the point of application of *R* and in the direction of *R*. Then draw the line *D* of a length equal to the perpendicular distance from *F* to line *c d*. This will be the true lever arm for *R*, and the moment of *R* will be the product of *R* and the distance *D*.

Since the moment of *P* measures the rotative effect of this force and the moment of *R* measures the rotative effect of the resistance, it is clear that if the lever is to balance, these two moments must be equal. If *L* is longer than *D*, as it is in this case, then *R* must be enough greater than *P* to make up for this, or otherwise the lever would begin to turn about *F*. This, in substance, is all there is to the principle of moments. The principle states that, if a body is to be in equilibrium, the sum of the moments of the forces which tend to turn it in one direction about a point is equal to the sum of the moments that tend to turn it in the opposite direction about the same point. In other words, if a body is to balance about a point, the opposing moments must be equal.

Calculation of Simple Levers

We will now be ready to solve examples of the lever by the aid of the principle of moments, and we will first consider that the weight

of the lever may be neglected, and that there are only two forces acting—the push or pull—which is applied to the lever, and the resistance overcome, these being balanced, of course, by the pressure at the fulcrum, which, in reality, is another force, but which need not be considered for the present, at least.

In Fig. 8 is shown a lever supported on the fulcrum F. At one end a push, P, of 10 pounds, is exerted, and at the other end is a resistance R, in the shape of a 100-pound weight. The distance from F to P is 40 inches, and from F to R, 4 inches. The principle of moments states that when a lever is in balance, the moment of the force tending to turn it in one direction must equal the moment of the force tending to turn it in the opposite direction. In Fig. 8 the moment of force P about fulcrum F, tending to depress the left-hand end of the lever, is $10 \times 40 = 400$ inch-pounds, and the moment of force R is $100 \times 4 = 400$ inch-pounds also, so that the lever is in balance.

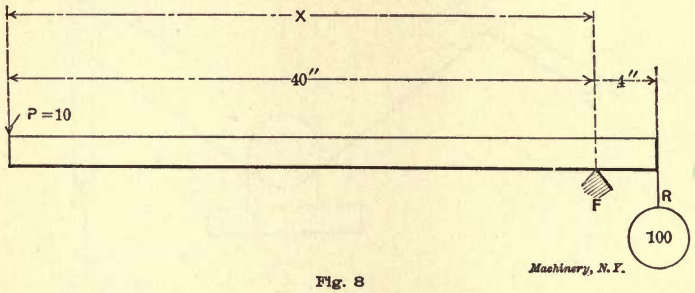


Fig. 8

Now, suppose that we had P, R, and the distance from F to R given in Fig. 8, and that we wanted to find the distance from F to P, which we will call x. By the principle of moments we have,

Moment of $P = 10 \times x$,

Moment of $R = 100 \times 4 = 400$.

But these moments are equal; hence, $10 \times x = 400$, and what we have to do is to find the value of x. It is clear that, if ten times the distance $x = 400$, the distance x must be 1/10 of 400, and all we have to do is to divide 400 by 10, giving 40 inches as the distance x.

Again, suppose it were desired to find the resistance R, the other quantities being known. For convenience we will take the moment of R first, because this contains an undetermined value. This is always a good rule to follow.

Moment of $R = 4 \times R$. (It makes no difference whether the 4 or the R is written first, but it is usual to write the figure first.)

Moment of $P = 10 \times 40 = 400$,

Then $4 \times R = 400$, and $R = \frac{400}{4} = 100$ pounds.

These simple examples contain all that need be known to solve lever problems where there are only two forces acting; but to make the subject still clearer, a more general example will be taken.

In Fig. 9 the lever shown is pivoted at *F*, which serves as the fulcrum. A push *P* is exerted by the rod at the right, which receives its motion from the cam and roller, as indicated. This push acts to overcome a resistance *R*, which acts along the rod seen at the left, and which may be supposed to consist of the resistance of the spring coiled around the rod, and of any piece of mechanism that this rod may have to operate. Let it be required to find how great a push, *P*, is necessary to overcome a resistance, *R*, of 250 pounds. The first thing is to find the length of the true lever arms, since without these the moments cannot be determined. To do this, first draw lines through the points on the lever at which the forces act, and in the direction in which they act. Thus, the force *P* acts at the point *C*, and the line *D H* indicates the position and direction of this force. Likewise the force *R*

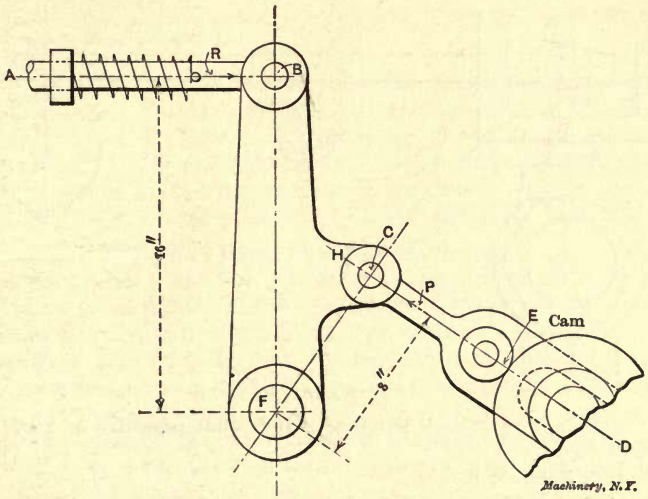


Fig. 9

acts at point *B*, and line *A B* indicates the position and direction of force *R*.

Now, the lever arm of force *P* is the perpendicular distance from *F* to line *D H*, and the lever arm of force *R* is the perpendicular distance from *F* to line *A B*. Assume that these distances measure 8 and 16 inches, respectively. Then,

Moment of $P = 8 \times P$.

Moment of $R = 250 \times 16 = 4{,}000$.

$$8 \times P = 4{,}000; \text{ and } P = \frac{4{,}000}{8} = 500 \text{ pounds.}$$

Example.—Suppose $P = 400$, $R = 150$, and the short arm $= 6$ inches. What is the length of the long arm? Answer—16 inches.

The safety valve in Fig. 10 is an example of a lever in which there are three forces to be considered, if we take into account the weight

of the lever, which is quite essential to do. The valve at V is acted upon by the pressure of the steam, tending to raise it. This pressure constitutes the push P upon the lever, which is resisted by the suspended weight R, and the weight of the lever, which we will call R_1. The weight of the lever is effective at the point G, the center of gravity of the lever. This point can be found by balancing the lever on a knife edge, the center of gravity being directly over the knife edge. The fulcrum of the lever is at F, and the lever arms for R, R_1 and P are marked A, B, and C, respectively.

Example 1.—Assume that $A = 30$ inches, $B = 14$ inches, $C = 3$ inches,

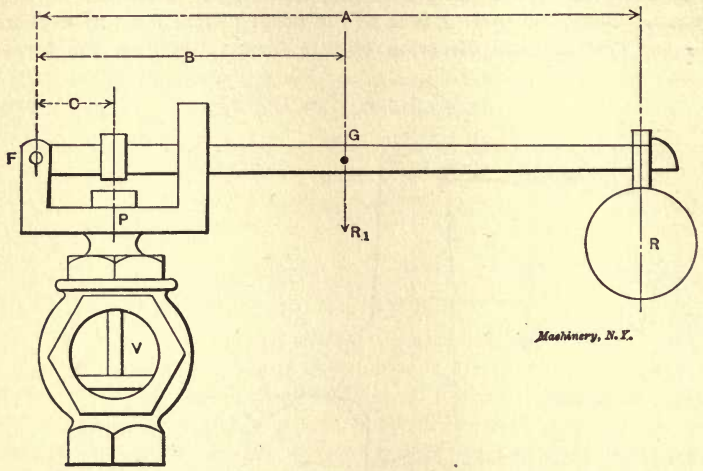


Fig. 10

$R = 20$ pounds, and $R_1 = 8$ pounds. Find what pressure of steam the valve will carry.

Moment of $P = 3 \times P$,

Moment of $R = 20 \times 30 = 600$,

Moment of $R_1 = 8 \times 14 = 112$.

For the valve to balance, the moment of P must be equal to the sum of the moments of R and R_1, for the moment of P tends to raise the lever, and the other moments tend to hold it down. Adding the moments of R and R_1, therefore, we have $600 + 112 = 712$, and this must balance the moment of P or $3 \times P$. Hence, $3 \times P = 712$, and $P = \frac{712}{3}$ $= 237\ 1/3$ pounds. This last part of the operation is like the work of the previous examples. The 237 1/3 pounds is the total pressure upon the valve, and to obtain the pressure per square inch that can be carried, we have simply to divide 237 1/3 by the area of the valve. To be theoretically exact, the weight of the valve and stem should be added to the figure 237 1/3.

Example 2.—Suppose it were desired to carry a total pressure upon the valve of 300 pounds. With the other dimensions remaining as

before, how heavy a weight R would have to be provided? Again, taking moments, we have,

$$\text{Moment of } R = 30 \times R,$$
$$\text{Moment of } R_1 = 8 \times 14 = 112,$$
$$\text{Moment of } P = 300 \times 3 = 900.$$

The sum of the first two moments must equal the last one, but we cannot add them as they stand, because we do not yet know what the first one is. Hence we will indicate the addition as follows:

$$30 \times R + 112 = 900.$$

Those who have had a little practice with formulas will have no trouble with finding the value of R; but for the benefit of those who have not, it can be said that we subtract the 112 from 900 and proceed as in the other examples. Thus, $900 - 112 = 788$, whence

$$R = \frac{788}{30} = 26\ 4/15 \text{ pounds.}$$

The following explanation will make the reason for subtracting 112 from 900 clear. We have found that the moment of R is 788; of R_1, 112; and of P, 900. Now, if 788 added to 112 equals 900, 900 must be 112 greater than 788, and 788 must be equal to 900 with 112 subtracted from it. Again, taking the formula as we have it, if $30 \times R$ plus 112 equals 900, $30 \times R$ must equal 900 with 112 subtracted from it.

Calculation of Compound Levers

It often happens that it is necessary to use two or more levers connected one to the other in a series, where it would not be convenient to obtain the desired multiplication with a single lever, or where it is necessary to distribute the forces acting. In such cases the levers are called compound levers, and their application is found in testing machines, car brakes, printing presses, and many other machines and devices. Probably the most familiar example is that of a pair of scales, and we will take this to illustrate the method of making the calculations for compound levers.

In Fig. 11 is a diagram showing an arrangement of levers that might be used for platform scales. The fulcrums of the various levers are in each case marked F. The scale platform is at E, bearing at each end on levers C and D, and loaded at the center with 1,000 pounds. A pressure of 500 pounds, therefore, is transmitted to lever C at a point 6 inches from the fulcrum, and 500 to lever D. As lever D is proportioned exactly the same as that part of lever C to the left of the center line of the weight—that is, as the distance from F to L in each case is exactly 4 feet, and the short arms are each 6 inches long—it follows that the final effect is the same as though the whole 1,000 pounds acted at a point 6 inches from the fulcrum F of the lever C.

Continuing through the various connections, the right-hand end of C pulls down on the lever B at a point 8 inches from its fulcrum, and this in turn pulls down on the scale beam A at a point 4 inches to the left of its fulcrum, and lifts the weight R. Question: What weight at R is required to balance the 1,000 pounds on the platform, assuming

that the system of levers is in balance so that there is no unbalanced weight to be considered? This is always provided for by a counterpoise on the scale beam.

The best way to solve any example of compound levers is to first determine the number of multiplications of each lever. Lever *A* has arms 40 and 4 inches long, and multiplies 10 times; lever *B* multiplies 4 times; and lever *C*, 25 times. Each lever multiplies in the same direction; that is, it tends to increase the force acting when we start at point *R*. Hence, the total multiplication is $10 \times 4 \times 25 = 1{,}000$, and

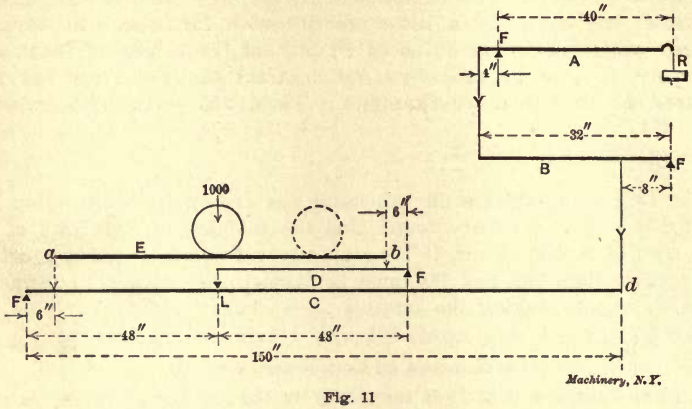


Fig. 11

thus one pound at *R* would balance the 1,000 pounds on the platform.

It may be asked whether with this arrangement the weighing of the scale would not be altered should the weight be moved to the dotted position shown in Fig. 11. A little thought will show that it would not. We have seen that the reduction from both points *a* and *b* to point *d* is 25 to 1, and it can make no difference whether 500 pounds acts at both *a* and *b*, or whether, for example, 300 pounds acts at *a* and 700 at *b*, the total 1,000 pounds being reduced 25 to 1 in either case.

CHAPTER IV

THE CENTER OF GRAVITY*

The force of gravity is exerted upon every one of the particles composing a body. The number of gravity forces acting upon a body may therefore be considered equal to the number of particles composing it. The sum or resultant of these individual forces constitutes the aggregate gravity of the body; and that point in the body at which may be applied a single resultant force that will have an effect the same as that of all the gravity forces acting upon its separate particles, is the center of gravity of the body. The center of gravity of a body will, therefore, be given by the position of the resultant of all the gravity forces acting upon its particles. If a body is supported upon its center of gravity, it will be in equilibrium in any position, and will have no tendency to rotate. This is, in substance, a definition that is sometimes given for the center of gravity.

Each one of the gravity forces acting upon the particles of a body, except those forces whose lines of action pass through its center of gravity, is producing a moment, and has a rotative effect. The lever arm of each moment is the perpendicular distance between the line of action of the force and the center of gravity of the body. Every such moment tends to produce rotation in the body, and as rotation is not produced when the body is supported upon its center of gravity, it follows that the center of gravity of a body is that point at which the moments of all the gravity forces acting upon its particles balance each other, or, in other words, at which the resultant moment of all the gravity forces is zero. This fact may be made use of in determining the position of the center of gravity. Different methods are employed for finding the center of gravity, according to the form of the body, or the arrangement of the system of bodies, for which it is to be found. Some of these methods will now be explained.

Center of Gravity of Lines

The word line, as here used, means a material line; that is, a homogeneous body of given length, having a uniform and very small transverse section, such as a fine wire. A theoretical line would, of course, have no width or thickness, and consequently, no mass and no gravity.

Single, Straight Line

The center of gravity of a straight line is at its middle point. If we conceive the line to be composed of uniform individual particles, the gravity of each particle will be the same; and the distance of each particle on one side of the middle point, from that point, will be the same as that of the corresponding particle on the opposite side.

* MACHINERY, September and October, 1898.

Hence, the moments of all the gravity forces acting upon the particles, taken about the middle point of the line, will balance, and that point will, therefore, be the center of gravity of the line. A straight line will balance upon its middle point; if supported upon that point, it will be in equilibrium in any position, and will have no tendency to rotate.

Two Straight Lines of Different Length

Let $A\ B$ and $C\ D$, Fig. 12, be two straight lines of any lengths and having any positions with respect to each other. The center of gravity of each line is at its middle point, as O and O_1. If these two centers of gravity be connected by the straight line $O\ O_1$ the center at gravity of the system will be somewhere on this line. Draw the line $O\ B_1$ equal and parallel to $O_1\ B = \frac{1}{2}\ A\ B$; on the opposite side of $C\ O_1$ lay off on the line $B\ A$, a length of $O_1 C_1$ equal to $O\ C = \frac{1}{2}\ C\ D$, and draw $B_1 C_1$. The point g, where the lines $O\ O_1$ and $B_1 C_1$ intersect, will be

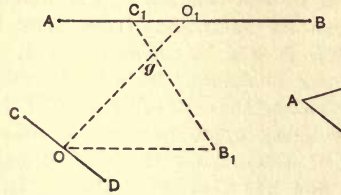


Fig. 12

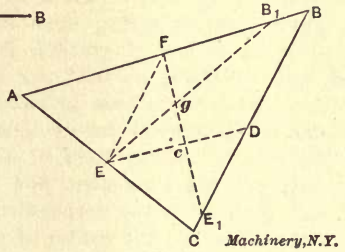


Fig. 13

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the center of gravity of lines $A\ B$ and $C\ D$. If the given lines are parallel, $O\ B_1$ is simply laid off on $O\ D$ prolonged. The distance $O_1\ g$ may also be calculated; it is given by the equation:

$$O_1 g = \frac{C\ D \times O\ O_1}{A\ B + C\ D}$$

Perimeter of the Triangle

Let $A\ B\ C$, Fig. 13, be any plane triangle, in which D, E and F are the centers of gravity of the three respective sides. Join any two of these centers, as D and E, and on this line determine, by the method just explained, the center of gravity c of the two sides joined. To do this, join E and F; the line $E\ F$ will be equal and parallel to $C\ D$; then lay off $D\ E_1$ equal to $C\ E$; the intersection c of the lines $D\ E$ and $E_1\ F$ will be the center of gravity of the sides $B\ C$ and $C\ A$. Now lay off $F\ B_1 = \frac{1}{2}\ A\ E + \frac{1}{2}\ B\ D$ and draw $E\ B_1$; the intersection g of the lines $E\ B_1$ and $c\ F$ will be the center of gravity of the three sides, or perimeter, of the triangle.

Circular Arc

Let $A\ B\ C$, Fig. 14, be the arc of a circle whose center is at O; $A\ C$ is the chord and B is the middle point of the arc. The center of gravity of the arc will be at some point g on the radius $O\ B$, at such distance from O that

$$Og = \frac{AC \times BO}{ABC}$$

Center of Gravity of Plane Surfaces

A theoretical surface has no thickness, and, therefore, no mass and no gravity. In mechanical problems, however, it is often necessary to find the center of gravity of a plane figure, or, more correctly, that point in its surface corresponding to what would be the center of gravity of the figure, were it a material body of uniform thickness. As here used, therefore, the word surface may be taken to mean a material surface, such as a very thin, homogeneous plate of a piece of cardboard.

Axis of Symmetry

If a plane figure can be divided by a straight line in such a manner that the two parts of the figure will exactly coincide when folded together along the line, the line so dividing the figure is called an

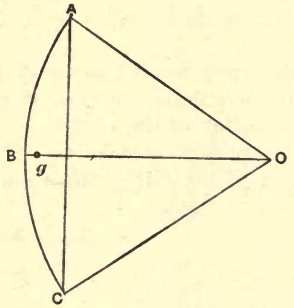

Fig. 14

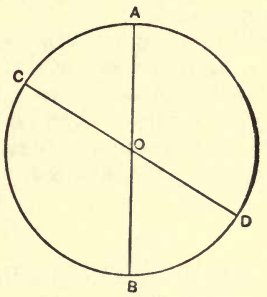

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Fig. 15

axis of symmetry. The diameter of a circle and the diagonal of a square are axes of symmetry for those figures.

The center of gravity of a plane figure having an axis of symmetry, must lie on such axis; if the figure has more than one axis of symmetry, the center of gravity must be at the intersection of the axes. Let *A B*, Fig. 15, be a diameter of a circle whose center is at *O;* it is also an axis of symmetry, for, if folded along this diameter, the two parts of the circle will exactly coincide. If, now, we consider the area of the circle to be composed of straight lines perpendicular to *A B*, which are not shown in the figure, the diameter *A B* will bisect each line; in other words, it will pass through the center of gravity of each line composing the area of the circle. Hence, the center of gravity of the entire system of lines composing the area of the circle, which will be the center of gravity of the circle itself, must be some point on the diameter *A B*. In like manner it can be shown that the center of gravity of the circle must lie on any other diameter, as the diameter *C D*. Consequently, the center of gravity of the circle must be at the center

O, the only point common to all diameters. That the center of gravity of the circle is at the geometrical center of the figure is so evident as to scarcely require proof; but the circle serves as a very simple example to illustrate the process of reasoning, which applies to any plane figure having two axes of symmetry, such as a circle, ellipse, rectangle, rhombus, equilateral triangle, square, or any regular polygon, and also to the perimeters of such figures.

Center of Gravity of Parts of Circles

Semicircle. The center of gravity is located on its axis of symmetry, at a distance of $0.4244r$ from the center of the circle, r being the radius of the circle.

Sector of a Circle. The center of gravity is located on its axis of symmetry, at a distance x from the center of the circle, the value of x being given by the equation:

$$x = \frac{2cr}{3l},$$

in which c is the chord and r the radius of the circle, and l the length of the arc.

Quadrant of a Circle. The center of gravity is located on its axis of symmetry, at a distance of $0.4244r$ from each radial side, or $0.6002r$ from the center of the circle, r being the radius of the circle.

Segment of a Circle. The center of gravity is located on its axis of symmetry, at a distance x from the center of the circle, the value of x being given by the equation:

$$x = \frac{c^3}{12a},$$

in which c is the chord and a the area of the segment.

Other Surfaces with Curved Outlines

Parabolic Surface. The center of gravity is located on its axis of symmetry, at 2/5 the length of the axis from the base.

Semi-parabolic Surface. The center of gravity is located at 2/5 of the length of the axis of the parabola from the base, and 3/8 the length of the semi-base from the axis.

Surface of a Hemisphere. The center of gravity is located at the middle of its axis or center radius.

Gravity Axis

It is not necessary, however, for a plane figure to have two, or even one, axis of symmetry, in order that its center of gravity may be determined. Any plane figure can be balanced upon a knife edge. The position of the knife edge will be defined by a straight line in such a position that the moments of all the gravity forces acting upon the particles composing the surface on one side of the line will just balance the moments of those on the other side. This line, about which the moments of the gravity forces balance, will here be called a gravity axis. By a process of reasoning analogous to that employed in finding

the center of gravity of the circle, it can be shown that every gravity axis of a plane figure contains the center of gravity of the figure. Consequently, the intersection of any two gravity axes determines the position of its center of gravity. It should be noticed that in many practical problems it is necessary to find the position of a gravity axis only, the exact center of gravity not being required.

Triangle

Let *A B C*, Fig. 16, be any triangle; the line *C D* extends from the vertex *C* to the middle of the opposite side. If we imagine the area of the triangle to be composed of straight lines parallel to the base *A B*, each of these parallel lines will be bisected by the line *C D;* that is, the line *C D* will pass through the center of gravity of each of the parallel lines. Every line composing the area of the triangle, and, consequently, the triangle as a whole, will just balance upon the line

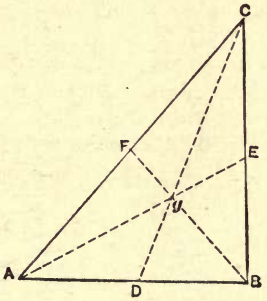

Fig. 16

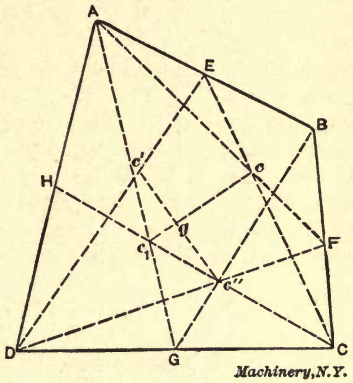

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Fig. 17

C D, which will be a gravity axis of the triangle. If, also, a line be drawn from any other vertex of the triangle to the middle of the opposite side, as the line *A E*, it will also be a gravity axis. As the center of gravity must lie on both these gravity axes, it must be at their intersection *g*. It is not necessary, however, to draw more than one gravity axis, in order to determine the position of the center of gravity of a triangle. If a line be drawn from any vertex to the middle of the opposite side, the center of gravity of the triangle will be on this line and at two-thirds the length of the line from the vertex. Thus, the center of gravity *g*, Fig. 16, is at two-thirds the length of *A E* from *A*, two-thirds the length of *B F* from *B*, and two-thirds the length of *C D* from *C;* its position may be located on any one of the lines.

Trapezium

There are several quite satisfactory methods for finding the center of gravity of a trapezium. The following simple method is probably as expeditious as any, and, as it depends upon the method just explained for finding the center of gravity of a triangle, and is readily

connected with that method, it has the advantage of being easily remembered.

Let *A B C D*, Fig. 17, be any four-sided plane figure. Consider it first to be divided into the two triangles *A B C* and *A D C*. The points *E*, *F*, *G*, and *H* are the centers of the respective sides, the common side *A C* not being drawn. The intersection *c* of the lines *A F* and *C E* is the center of gravity of the triangle *A B C*, and, similarly, the intersection c_1 of the lines *A G* and *C H* is the center of gravity of the triangle *A D C*. The line $c\,c_1$ connecting these two centers of gravity, will be a gravity axis of the entire figure. The trapezium is then considered to be divided into the triangles *B A D* and *B C D*, and, by a similar construction, the position of the gravity axis $c'\,c''$ is determined. The intersection *g* of these two gravity axes will be the center of gravity of the trapezium.

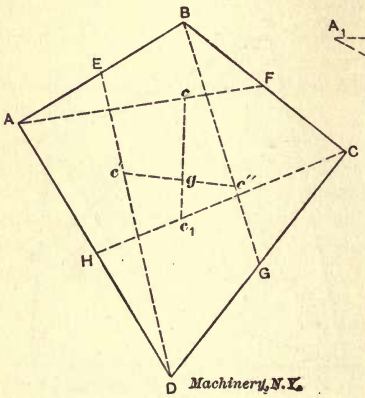

Fig. 18

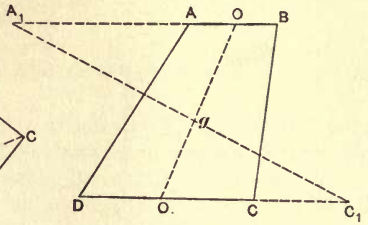

Fig. 19

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For this construction, it is not necessary to draw the entire portion of each constructional line, as shown in the figure, but only such portions of the lines as are necessary to locate their intersections. Some may prefer the construction shown in Fig. 18; it is the same as that shown in Fig. 17, except that only one gravity axis is drawn for each triangle, and the center of gravity of the triangle located at two-thirds the length of the axis from its vertex.

Trapezoid

If the figure is a trapezoid, the following construction, taken from "Trautwine's Engineer's Pocket Book," is a very simple method of finding its center of gravity. Let *A B C D*, Fig. 19, be any trapezoid for which the center of gravity is to be found. Prolong the two parallel sides in opposite directions, making each prolongation equal to the other side, and join the extremities of the prolongations by a straight line; also join the centers of the parallel sides. The intersections of these lines will be the center of gravity of the figure. Thus, in the figure, $A\,A_1$ is made equal to *D C*, and $C\,C_1$ equal to *A B*, and the

extremities of the prolongations joined by the line A_1C_1, while the line $O\,O_1$ joins the centers of the parallel sides; the intersection g of the lines A_1C_1 and $O\,O_1$ is the center of gravity of the trapezoid.

Irregular Figure

The center of gravity of any irregular figure bounded by straight lines may be found by dividing it into triangles, finding the center of gravity of each triangle, and then finding the center of gravity of the system of triangles, the area of each being considered to be concen-

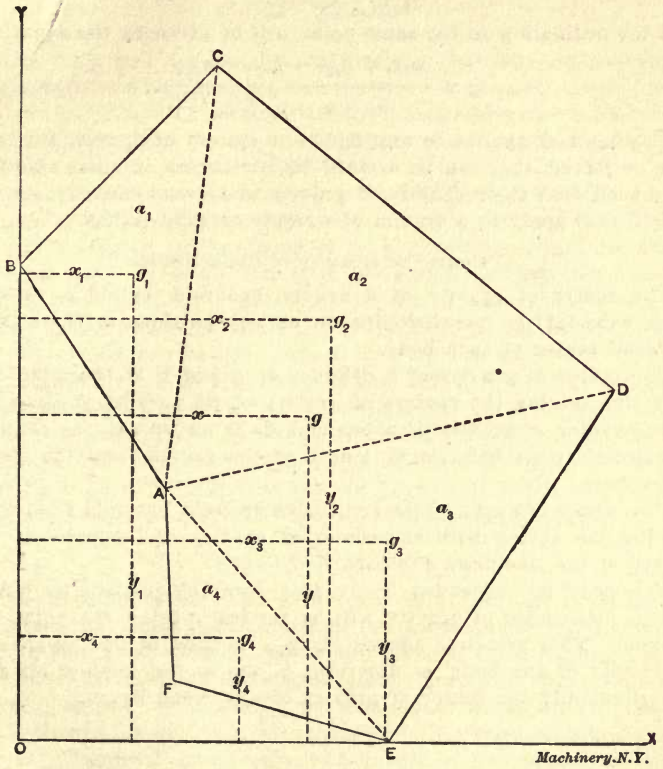


Fig. 20

trated at its center of gravity. For finding the center of gravity of the system of triangles, the method of rectangular co-ordinates may be employed. Let $A\,B\,C\,D\,E\,F$, Fig. 20, be any irregular figure bounded by straight lines. By the lines $A\,C$, $A\,D$, and $A\,E$ the figure can be divided into the four triangles $A\,B\,C$, $A\,C\,D$, $A\,D\,E$, and $A\,E\,F$, whose centers of gravity, g_1, g_2, g_3, and g_4 may be found by the method explained for triangles. Draw the vertical and horizontal axes $O\,Y$ and $O\,X$, intersecting at O; these may be any vertical and horizontal lines, but it is generally convenient to draw them through the left-hand and lower extremities of the figure, as shown; $O\,X$ is the axis of

abscissas and OY is the axis of ordinates. The lines x_1, x_2, x_3, and x_4 are, respectively, the abscissas of the centers of gravity from the axis of ordinates; and the line y_1, y_2, y_3, and y_4 are, respectively, the ordinates of the same points, or their perpendicular distances from the axis of abscissas. If a_1, a_2, a_3 and a_4 represent the areas of the four respective triangles, then the abscissa x to the center of gravity g of the entire figure will be given by the equation:

$$x = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4}{a_1 + a_2 + a_3 + a_4},$$

and the ordinate y to the same point will be given by the equation:

$$y = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4}{a_1 + a_2 + a_3 + a_4}.$$

This method applies to any figure or system of figures, either separate or joined, that can be divided into triangles or other simpler figures such that their centers of gravity and areas can be determined. It will also apply to a system of weights or solid bodies.

Center of Gravity of Solid Bodies

The center of gravity of a sphere, spheroid, cylinder, cylindrical ring, cube, prism, parallelopipedon or any polyhedron is at the geometrical center of each body.

The center of gravity of a cylinder or prism is at the middle point of a line joining the centers of gravity of its parallel surfaces.

The center of gravity of a hemisphere is on its axis, or radius perpendicular to its base, at $\frac{3}{8}$ length of the radius from the center of the sphere.

The center of gravity of a right cone or right pyramid is in the line joining the vertex with the center of gravity of the base, at $\frac{1}{4}$ the length of the line from the base.

If a body be suspended freely at a point other than its center of gravity, its center of gravity will be vertically below the point of suspension. This principle affords an easy method of finding the center of gravity of any body, as described in the second method for finding experimentally the center of gravity of any plane figure.

CHAPTER V

THE FIRST PRINCIPLES OF THE STRENGTH OF BEAMS*

Having mastered the written engineering language sufficiently to deal successfully with formulas, the next step is to make the acquaintance of such engineering terms as are most frequently met with. Foremost among these are the terms relating to the strength of materials, and more especially the strength of beams.

If a bar is laid across two supports as in Fig. 21, and a weight placed in the center of it, we shall, if the bar be limber, witness the bending of the bar as shown, or as expressed in engineering terms, the deflection of the bar. It is obvious that the stiffer the bar, the less the deflection, and that a bar might be so lacking in stiffness as to actually break when the weight is placed upon it. Now the bar may lack stiffness from one or two causes; it may be that its dimensions are not well proportioned, or it may be made of soft and pliable materials. Sometimes both these causes are combined in the same bar. If the bar does not break when the weight is placed upon it, we must admit three facts; first, that the weight bends the bar; second, that the bar resists the bending; third, that the bar is able to resist the bending because it is large enough and made of stiff enough material.

Important Definitions

The bending effect that the weight has upon the bar is called the *bending moment* upon the bar due to the weight. The ability of the bar to resist the bending is called the *moment of resistance* of the bar. How these names first came into use the author does not know; perhaps there is no explanation, but the reader must not confuse the terms with any period of time because of the word moment. Time has nothing whatever to do with the strength of the bar, or the effect of the load upon it, except for such materials as wood, when loaded near to the limit of endurance.

In Fig. 21, the point at which the greatest bending occurs is directly under the weight, and we say the bending moment is maximum at this point, and the moment of resistance of the bar must equal the *maximum bending moment* at this point in the bar. In using the term bending moment, the engineer usually means the maximum bending moment, because this has the greatest bending effect upon the bar, and we shall hereafter drop the word maximum.

* MACHINERY, November, 1905.

Relation between Bending Moment and Moment of Resistance

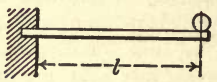
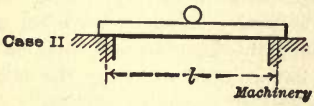
If now we let M = the bending moment on the bar, and R = the moment of resistance of the bar, we can express the relation of the two as given above thus:

$$M = R \qquad (33)$$

We said that the maximum bending moment was under the weight, and if the weight is placed further along on the bar, nearer one support than the other, the maximum bending moment will move with the weight. Also, if the bar is differently supported, the maximum bending moment will be at another point. For all cases of frequent occurrence, engineers have tables of formulas giving the position and amount of the maximum bending moment, so that it is only necessary to find in the tables the same case as the one we are considering, and

TABLE 2. BENDING MOMENT OF BEAMS UNDER VARIOUS SYSTEMS OF LOADING

W = total load.
l = length of beam in inches.
I = moment of inertia.
Z = section factor.

Case I 	Beam fixed at one end and loaded at the other. Max. bending moment at point of support = $W l$.
Case II 	Beam supported at both ends. Single load in middle. Max. bending moment at middle of beam $\frac{W l}{4}$.

Machinery

taking the formula there given, substitute for the letters the corresponding dimensions in our case, and we have a numerical expression for the bending moment. The formulas given in these tables consist of combinations of dimensions measured along the bar, and weights of the loads on the bar. If, when substituting values for letters in the formula, loads are taken in tons, and distances in feet, the bending moment will be expressed in foot-tons, while if loads are taken in pounds and distances in inches, the usual custom, the bending moment will be expressed in inch-pounds.

Table 2 is a small portion of such a table as may be found in any book on machine design in any drafting-room or factory, as well as in all the handbooks issued by the steel mills.

So much for the first member of our equation, the bending moment on the bar. We have already seen that the bar offers resistance to bending by reason of two things: its dimensions, and the character of its material, and we should expect to find both dimensions and materials accounted for in the formula for the moment of resistance of any bar. This is just what the formula for the moment of resistance does. It is composed of two parts or terms, one of which expresses the resisting effect of the material of the bar, and the other

expressing the resisting effect possessed by the bar because of its shape and size. Let us investigate each term by itself, taking first the resisting effect of the material.

Tension and Compression Stresses

Let the reader take an ordinary rubber eraser of the form shown in Fig. 22, and bend it as shown in Fig. 23. While holding the eraser in the best position, draw a sharp knife across the top side. The cut immediately spreads out in the form of a V as shown at *a*. Draw the knife a second time through the same cut and the V spreads a little more. Now draw the knife across the bottom. The cut immediately closes up as at *b*. Draw the knife a second time across the same cut and it will still close up completely. In making the second cut on this side it may be necessary to release the eraser from the bent position, because the closing cut grips the knife blade and makes cutting difficult, but the cut will close, upon again bending the rubber.

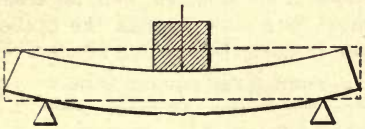


Fig. 21

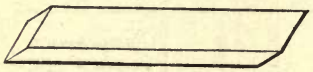


Fig. 22

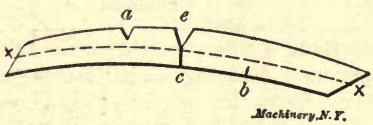


Fig. 23

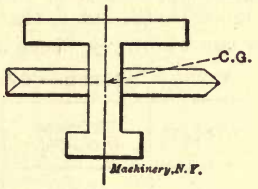


Fig. 24

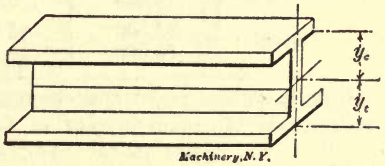


Fig. 25

Figs. 21 to 25

Having made the two cuts *a* and *b*, reverse the bend in the eraser and witness the closing of cut *a* and the opening of cut *b*. Now if you are a careful experimenter, you can start two such cuts as *a* and *b* directly opposite each other, and by cutting each one the same amount each time, you can succeed in bringing them nearly together in the center as shown at *c*. Of course, it will be impossible to bring them quite together, because that would cut the eraser apart, but by a little care you can satisfy yourself of these facts: that the portion of the eraser above the center line *x x* separates when cut; and that the portion below the line closes when cut. Reversing the bend of the eraser as before reverses the behavior of the cuts, but observe that whichever way the eraser is bent, the opening cuts are to be found on the convex side, and the closing cuts on the concave side.

We know that all material (engineering and building material at least) is composed of fibers, and we must conclude from the behavior of our eraser that all the fibers on the convex side of the line *x x*

are stretched when the eraser is bent, while the fibers on the concave side of $x\,x$ are compressed. Since the cut through the stretched fibers opens like a V, we may conclude that those fibers lying at the top of the V are stretched more before the cutting than those lying at the point of the V. A careful examination of the cut made through the compressed fibers will show that at the outer portion of the cut, the edges are raised slightly, while at the inner portion, near the center of the eraser, the edges are not raised. We can account for this only by assuming that the fibers at the outer portion are more compressed than those near the center of the eraser.

Having performed these experiments and noted the results, we must admit the following facts: 1st, that half the fibers of a bent bar are in compression while the other half are stretched, or, as engineers say, are in tension; 2nd, that the amount of compression or tension is greatest at the outer portion of the bar, and diminishes towards the center of the bar; 3rd, that it follows from this, as well as from experiments with cut *c*, that there must be a line through the center of the bar where the fibers are neither in compression nor tension.

TABLE 3. STRENGTH OF MATERIALS—POUNDS PER SQUARE INCH.

MATERALS.	Ultimate Strength.	Safe Working Strength.	Factor of Safety.	Kind of Stress.
Cast Iron.	88,640	16,000	5	Compression
	15,620	3,000	5	Tension
Steel.	82,500	16,000	5	Compression
	80,000	16,000	5	Tension

Now the fibers resist any change in their condition, either stretching or compressing, the amount of resistance differing in different materials. Iron fibers more than rubber fibers for instance. When a bar is bent, engineers speak of the fibers as being under stress, some being under compressive stress, and others under tensile stress, as we have seen, and they speak of the bar as being subjected to fiber stress. Now, fiber stress is expressed in pounds per square inch, and it is the duty of the engineer when designing a beam or other structure to keep the fiber stress within safe limits, and these safe limits are given in hand books for a great variety of materials, in tables of which Table 3 is a sample.

Factors Determining the Moment of Resistance

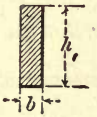
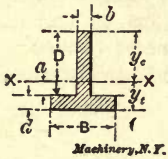
Since the material composing the bar derives its ability to resist bending by reason of the resistance of its fibers to changes, the fiber stress must be one of the terms expressing the moment of resistance of the bar. The fiber stress is denoted by the symbol f. The second term of the moment of resistance of the bar takes into consideration the strength the bar derives from its dimensions.

Bend the eraser in the direction of its greater thickness. We note it takes a much greater force to bend it thus than to bend it as we did at first, in the direction of its least thickness. If we repeat the

experiments with the cuts while bending the eraser thus, we shall find that everything witnessed before holds good for this case also. If we look for a reason for the greater force required to bend the eraser in the direction of its greater thickness, we shall find it in the fact previously observed, that the fibers are more stretched or compressed the further they are from the center line $x\,x$, and thus they present greater resistance to bending. The line $x\,x$ is called the *neutral axis*, because on it the fibers are neutral, being neither stretched nor compressed, and the fibers at the outer portion of the bar are called the extreme fibers, because they are furthest removed from the neutral axis $x\,x$.

The second term of the moment of resistance, taking account of the shape and size of the bar, is called the section factor, sometimes also

TABLE 4. VALUES OF I (MOMENT OF INERTIA) AND Z (SECTION FACTOR) FOR VARIOUS SECTIONS.

Section.	I	Z	Area.
Case I. 	$\frac{b\,h^3}{12}$	$\frac{b\,h^2}{6}$	$b\,h$
Case II. 	$\frac{b\,y_c^3+By_t^3-(B-b)a^3}{3}$	$\frac{I}{y_t}$ $\frac{I}{y_c}$	$Bd+bD$

called the section modulus, Z, and is given in all hand books in the shape of tables for different shapes of beams, in the style shown in Table 4.

The neutral axis $x\,x$ is not always in the center of the bar, but it always passes through the center of gravity of the cross section of the bar.

Center of Gravity

Here we shall have to digress for a moment, since it is the intention to leave no term unexplained. The reader may best become acquainted with the center of gravity in the following manner: Cut out of stiff cardboard the shape of the cross section of the bar, and balance it over a sharp edge, in a manner as shown in Fig. 24. Draw a line across the card corresponding to the edge over which it is balanced. Repeat the experiment, turning the card around on the edge, and, balancing it a second time, draw another line. The intersection of these two lines will be the center of gravity of the section of the beam. If the experiment has been done with sufficient care, the card may be balanced upon a sharp point placed at the intersection of the two lines, just as if the entire material of the card were placed vertically above the point. A definition frequently met with is: The center of gravity is that point at which the entire weight of a body may be considered as concentrated.

Another way of finding the center of gravity is to suspend the card by a fine thread alongside of a plumb line, and when the card and line have come to rest, mark the position of the plumb line on the card. Turn the card around, and suspend a second time from a different point, and mark the position of the plumb line again. Where the two marks of the plumb line cross will be the center of gravity of the figure. No matter from how many points the card may be suspended, the plumb line will always be found to pass through the center of gravity. A line in the center of the beam directly opposite the center of gravity thus found will be the neutral axis.

Equation for Bending Moment

If we now take the equation expressing the relation of the bending moment on the bar to the moment of resistance of the bar, and use the symbols for the two parts of the moment of resistance, we shall have

$$M = R = f Z.$$

Some tables do not give the section factors Z for all sections directly, but say it is $\frac{I}{y}$, and therefore we must understand this expression.

The denominator y of the fraction is the distance from the neutral axis $x\,x$ to the extreme fiber of the bar, see Fig. 25, and the numerator I is what is called by engineers the *moment of inertia* of the section of the bar. Here again there is a chance for confusion because of the use of the word inertia.

Moment of Inertia

The term moment of inertia was originally employed when comparing the energies of rotating bodies. We know that a moving body possesses energy due to that property of matter which engineers call inertia. Inertia is not a force; it is simply resistance, and is due to the incapability of a dead body to move, or of a moving body to change its velocity or direction without the application of some external force. Now the number of foot-pounds of energy possessed by a moving body is equal to $\frac{1}{2} M V^2$, where M is the mass of the body, and V its velocity in feet per second. A moving body then, must be acted upon by an external force before it can be brought to rest. A rotating body is simply a very large number of particles moving in circular paths about an axis called the axis of rotation. Each moving particle, therefore, possesses energy due to its inertia, and the energy of each particle is equal to $\frac{1}{2} m v$, where m is the mass of the particle, and v its velocity in feet per second. But the energy varies as $m v^2$, because simply dividing by 2 does not change the relative values. It is also obvious that the circumferential velocity of each particle varies as the distance from the axis of rotation, which distance or radius we call r. Hence, substituting r for v, the energy of each particle varies as $m r^2$. Suppose we imagine that the whole mass of the rotating piece, that is, the sum of all the small particles m, is concentrated in a circle that is of such diameter that the energy pos-

sessed by the entire mass is the same as before. The radius of this imaginary circle is called the radius of gyration, and is usually designated by the letter r. Now we may say that Mr^2, where M stands for the whole mass, is a measure of the energy of the rotating piece. This expression Mr^2 is given the name moment of inertia, each particle of which the rotating body is composed possessing a turning moment about the axis of rotation, due to its motion and inertia.

When it was discovered that the flexure of a beam depended upon the value ar^2 (where a is the area of the cross section of the bar, and r^2 is the mean of the squares of the distances of the infinite number of small areas into which the area of the section may be supposed to be divided, from the center of gravity of the section) ar^2 was seen to be similar to the expression Mr^2, which, in connection with the rotating bodies, had already become known as the moment of inertia; so, very carelessly on the part of those who first committed the error, it was said that the flexure of a beam varied as its moment of inertia, not because inertia has anything to do with it, for, of course, it has not, but because ar^2, the expression for the moment of resistance to flexure, happened to vary in the same way as the moment of inertia Mr^2 of the same body when rotating about its center of gravity.

The moment of inertia of a bar may be calculated by several methods, but the table in hand books give it for all usual shapes of sections, and we will not attempt the calculation here.

Universal Formula for Bending Strength of Beams

Since we are sometimes able to find in tables only the moment of inertia of a bar, and not the section factor, we must bring our formula one step further, thus:

$$M = R = fZ = f\frac{I}{y} \tag{34}$$

or

$$Z = \frac{I}{y} = \frac{M}{f} \tag{35}$$

and here we have the formula for determining the size required for any beam.

For beams in which the center of gravity is not the center of the beam, there will be two values of y, one of which we will denote as y_c, being the distance from the neutral axis to the extreme fibers in compression, and the other as y_t, being the distance from the neutral axis to the extreme fibers in tension, see Fig. 25.

In some materials the ability of the fibers to resist tension is about equal to their ability to resist compression, while in other materials there may be great inequality in this direction, some being much stronger in tension than in compression, while others are stronger in compression than in tension. In such a material we shall have two values of f, which we will denote as f_c and f_t for compression and tension, respectively.

Ultimate and Safe Stresses

Some tables on the strength of materials give what is called the ultimate or breaking strength of the materials, while other tables give the safe working strength of materials.

When using the latter tables, the values given are to be substituted directly for f_c and f_t in the formula. Since it would not do to have the material of which a beam is made strained up to the breaking point, we must, when using the former tables, make use of a factor of safety. This factor of safety is a divisor by which the breaking strength of a beam is divided to allow a margin of strength in the beam. The divisor varies from 2 to 10, and the proper use of different divisors is given in the text books.

To illustrate, the breaking strength of steel may be given as 80,000 pounds per square inch, and $\frac{80{,}000}{5} = 16{,}000$. If we substitute 16,000 for f in the formula, we shall be working out our results with a factor of safety of 5, and the beam should not actually break until loaded with five times the load designed for. As a matter of fact, the beam would become badly bent long before five times the load could be placed upon it.

Limit of Elasticity

We have seen that all material deflects under the influence of a load, and we suppose that the elasticity of the material causes it to spring back to its original condition when the load is removed. This is true within limits, but there is a point somewhere between the safe load and the breaking load at which, when the load is gradually increased, the beam becomes strained beyond its power to return to its original condition upon the load being removed. This point is variously called the *limit of elasticity*, the *yield point*, the *point of permanent* set.

Practical Examples

Let us now take up two examples illustrating the ground we have just passed over, and the use of the tables.

Example 1. A rectangular steel bar, 2 inches thick, is built into a wall as in Fig. 26, and is to hold a load of 3,000 pounds at its outer end, 36 inches from the wall. We wish to know the required depth to make the beam.

1st. Consider the bending moment on the beam. According to Case 1, Table 2, the bending moment is

$$M = W l.$$

For our case we know W and we know l, and substituting these for the letters in the formula gives us

$$M = 36 \times 3{,}000 = 108{,}000 \text{ inch-pounds.}$$

2d. Consider the permissible fiber stress in the steel bar. Table 3 gives the safe working strength of steel as 16,000 pounds per square inch.

3d. Using Formula (35) we can find the value of the section factor for our beam. We know the bending moment and we know the fiber

stress, so substituting these for the letters in the formula we get

$$Z = \frac{M}{f} = \frac{108{,}000}{16{,}000} = 6.75.$$

4th. Find the section of our beam in Table 4, Case 1, where we find that the section factor is

$$Z = \frac{b\,h^2}{6}.$$

We know Z and we know b, so substituting these values for the letters, we get

$$6.75 = \frac{2 \times h^2}{6}.$$

If we multiply both sides of this equation by 6, we shall not change its value, but shall get

$$6 \times 6.75 = 2 \times h^2.$$

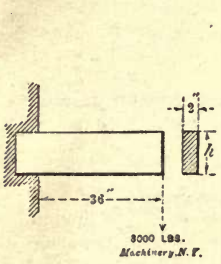


Fig. 26

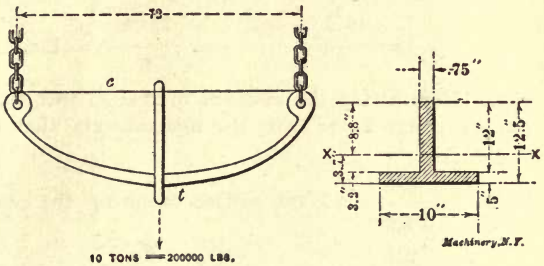


Fig. 27

If we now divide both sides by 2, we shall not change its value, but shall get

$$\frac{6 \times 6.75}{2} = h^2 = 20.25.$$

5th. We can most conveniently find the square root of 20.25 from a table of squares and roots which may be found in any hand book. This square root is 4.5, and we thus find that

$$h = 4.5 \text{ inches.}$$

If we make the beam 2 inches thick by 4.5 inches deep by 36 inches long, it will support a load of 3,000 pounds at its free end, and the fibers will be strained to 16,000 pounds per square inch.

Example 2. Let us undertake to design a suspension beam like Fig. 27 to carry ten tons, the material to be cast iron. The proposed section of the beam is more complicated than that of the previous example, and we cannot obtain a result quite so directly.

1st. Inspect the proposed beam to locate the compression and tension flanges. We find the compression flange is on top and the tension flange on the bottom, and we mark them c and t respectively.

2d. Table 3 shows us that cast iron is stronger in compression than in tension, hence we conclude that we should have more metal on the

tension side than on the compression side, and accordingly we place the section with the heavy side down.

3d. Assume a section by making the best guess possible as to the dimensions shown heavy in the figure. Cut out this section of cardboard, and find the location of the neutral axis $x\,x$ as previously explained. Now fill in the figures shown light by measuring the cardboard section.

4th. Find the section in Table 4. Here we find that before we can get the section factor of the beam we must get the moment of inertia of the beam. Substitute the dimensions of our section for the letters of the formula given in Table 4, and we shall get

$$I = \frac{(0.75 \times 8.8^3) + (10 \times 3.7^3) - (10 - 0.75)\ 3.2^3}{3}$$

$$= \frac{511.1 + 506.5 - (9.25 \times 3.2^3)}{3}$$

$$= \frac{1017.6 - 303.12}{3} = \frac{714.48}{3} = 238.$$

5th. Now divide the moment of inertia just found by the distances of the extreme fibers from the neutral axis, that is, by y_c and y_t, and we get

$$Z_c = \frac{I}{y_c} = \frac{238}{8.8} = 27,$$ the section factor for the compression side.

$$Z_t = \frac{I}{y_t} = \frac{238}{3.7} = 64.3,$$ the section factor for the tension side.

6th. Inspect Table 2 and find the bending moment on the beam according to Case 2; substituting the dimensions of the beam, and the load to be carried, in the formula given, we have

$$M = \frac{W\,l}{4} = \frac{20{,}000 \times 72}{4} = 360{,}000 \text{ inch-pounds.}$$

7th. Dividing the bending moment just found by the section factors found in the 5th step, will give the fiber stress on the beam according to Formula (35), thus

$$\frac{360{,}000}{27} = 13{,}333$$ pounds per square inch on the compression side.

$$\frac{360{,}000}{64.3} = 5{,}600$$ pounds per square inch on the tension side.

The latter is too high, so another guess must be made, making the section heavier on the tension side. Then the steps 3, 4, 5 and 7 must be repeated, and if the fiber stress then comes below 3,000 pounds per square inch, the section will be right.

CONTENTS OF DATA SHEET BOOKS

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